

COURSE

How Lighthouses Work

9 Chapters

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HOW LIGHTHOUSES WORK

Contents

Chapter 1	Light and Distance	2
Chapter 2	Early Beacons	10
Chapter 3	Lamps and Fuel	20
Chapter 4	Lenses	31
Chapter 5	Rotation and Flashes	40
Chapter 6	Building on Rock	49
Chapter 7	The Stevensons	59
Chapter 8	Fog and Radio	70
Chapter 9	Plan a Light	82

How Lighthouses Work 🏆

Chapter 1 · Light and Distance

A lighthouse is a tower built to carry a light, and its job is the oldest one in navigation: to tell a sailor where the land, the rock or the harbour mouth is. The Wikipedia article on lighthouses describes one as a structure designed to emit light as a navigational aid, marking dangerous coastlines, shoals, reefs and rocks, and marking safe entries to harbours. Before any question about lenses, lamps or stone, there is a question of physics: **how far away can a light be seen?** The answer has three parts, and a lighthouse designer has to keep them apart. How far the light's strength carries it. How much the weather takes away. And how far the curve of the Earth allows it to be seen at all.

> **WHAT WAS RUN, AND WHAT TO TRUST**
>
> Every Python snippet in this chapter and in the three exercise solutions was executed, and the output in the comments is what Python printed. The definitions of the three ranges, the horizon constants and the inverse square law come from the sources named in the chapter. The *light intensities*, the *heights* and the *visibilities* used in the calculations are illustrative assumptions, labelled where they appear: they show how the principle works, and are not the figures of any real lighthouse. The later chapters bring in the real Scottish lights.

Three Questions, Three Ranges

RANGE	WHAT LIMITS IT	DEPENDS ON
Luminous range	the light becomes too faint for the eye, because it spreads out and the air absorbs it	intensity of the light, the clearness of the air
Geographic range	the curve of the Earth hides the light below the horizon	height of the light, height of the observer's eye
Nominal range	the luminous range in one agreed standard weather, so lights can be compared	intensity only

(Sources: the definitions agree across the US Lighthouse Society glossary, which I read directly, and the Boatnews article on lighthouse range, which I also read directly. The Britannica article on lights was blocked when I tried to fetch it, so what I know of it is a search summary only.) The

glossary defines the nominal range as the "luminous range of a light in nautical miles when the meteorological visibility is 10 nautical miles". Boatnews says the luminous range depends on the power of the source and the weather, while the geographic range depends on the Earth's curvature and the heights of the light and the observer. What a sailor really sees is the **smaller** of the luminous and the geographic range, and this chapter builds the tools to calculate both.

Light Spreads Out: The Inverse Square Law

A small lamp sends its light out in all directions. Move twice as far away and the same light is spread over a sphere with four times the area, so each patch receives a quarter as much. This is the **inverse square law**: the intensity of light from a point source is inversely proportional to the square of the distance (stated, for example, in the Wikipedia article on the law). The strength of a lamp in a given direction is its **luminous intensity**, measured in candela (cd), and the light it delivers to a surface facing it is the **illuminance**, in lux (lx), which is simply `E = I / d²` when the distance is in metres.

```
def illuminance(intensity_cd, distance_m):
    """Inverse square law: lux received on a surface facing the light."""
    return intensity_cd / distance_m ** 2

I = 1_000_000                      # assumed intensity, candela (illustrative)
for km in (1, 2, 4, 8):
    print(km, "km", illuminance(I, km * 1000), "lx")
print(illuminance(I, 2000) / illuminance(I, 1000))
# 1 km 1.0 lx
# 2 km 0.25 lx
# 4 km 0.0625 lx
# 8 km 0.015625 lx
# 0.25
```

Each doubling of the distance divides the light by four: 1.0, 0.25, 0.0625, 0.015625 lx. The last line confirms the ratio of 0.25 between 2 km and 1 km. This is the first reason a light cannot be seen forever. In empty space it would still weaken without limit, and the eye can only detect a light above some threshold. Doubling the range of a light means quadrupling its intensity, which is expensive, and it is why lighthouse engineers spent so much effort (as the later chapters show) gathering light into a beam instead of letting it spread in every direction.

The Air Takes Some: Allard's Law

Real air absorbs and scatters light, so the light is also weakened by a factor that falls exponentially with distance. Allard's law combines both effects: $E = I \cdot T^D / d^2$, where T is the fraction of light that survives each nautical mile and D is the distance in nautical miles. A search for the IALA (the international lighthouse authorities' body) documents turned up Allard's law as the standard way to calculate illuminance at the eye from intensity, distance and an exponential extinction factor, and the nominal range as the distance at which the illuminance falls to 0.2 microlux (2×10^{-7} lx) in a meteorological visibility of 10 nautical miles. I did not read the IALA text itself, only the search summary, which also notes that IALA does not regard 0.2 microlux as the true threshold of detection but as a convenient level above it. Treat 0.2 microlux here as the convention it is.

Meteorological visibility is the distance at which a dark object is just visible against the sky, which corresponds to 5% of the light getting through. That gives $T = 0.05^{1/V}$ for a visibility of V nautical miles. For 10 nm this is 0.741, which matches the "74 percent transmission" that the Britannica search summary quotes for the standard weather. (The 5% figure is my own knowledge, used here because it reproduces the 74%.)

The Earth Curves Away: The Horizon

Even a very bright light disappears when the Earth's curve hides it. For an eye at height h above a smooth sea, the geometry of a tangent line to a sphere of radius R gives a horizon distance of $d = \sqrt{2Rh}$ when h is small compared with R. With R = 6,371 km and h in metres this is **3.57 √h** kilometres (verified on the Wikipedia article on the horizon). The air bends light slightly downward, which lets you see a little further: under standard conditions the article says the factor becomes about **3.86**, an increase of about 8%. It also gives a "7/6 Earth radius" model of refraction.

```
import math

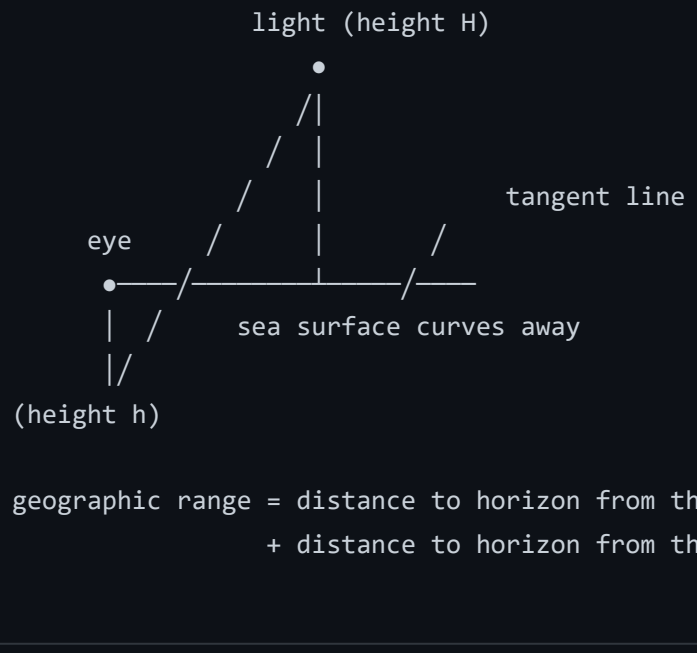
R = 6371.0                                  # mean Earth radius, km

def horizon_km(h_m, k=1.0):
    """Distance to the horizon for eye height h_m metres.
    k = 1 is the pure geometry; k = 7/6 is the 'Young' refraction model."""
```

```
    return math.sqrt(2 * (R * k) * h_m / 1000)

for h in (2, 10, 30, 60):
    print(h, round(horizon_km(h), 2), round(horizon_km(h, 7/6), 2))
print(round(3.57 * math.sqrt(30), 2), round(3.86 * math.sqrt(30), 2))
print(round(3.86 / 1.852, 3))
# 2 5.05 5.45
# 10 11.29 12.19
# 30 19.55 21.12
# 60 27.65 29.87
# 19.55 21.14
# 2.084
```

The columns are the height (m), the horizon without refraction (km) and the horizon with the 7/6 model (km). An eye 2 m above the sea sees only about 5 km, while one at 30 m sees 19.55 km, or 21.12 km with refraction. The two refraction estimates (21.12 km from the 7/6 model and 21.14 km from the constant 3.86) agree to 0.02 km, so they are the same idea in two forms. Dividing 3.86 by 1.852 km per nautical mile gives 2.084 nautical miles per square root of a metre. Sources disagree slightly on this constant: Boatnews prints 2.10, my arithmetic gives 2.084. Both are close to the "2.08" often quoted, and the gap is under 1%, but it is a reminder that the rules of thumb are approximations of an atmosphere that varies from day to day.



That is the key to the **geographic range**. A light of height `H` is visible over its own horizon distance, and an observer's eye adds its own, so the two distances simply add: `3.86 (√H + √h)` km. In nautical miles this is about `2.08 (√H + √h)` with the heights in metres. Notice that the range grows with the *square root* of the height, so a tower four times as tall sees only about twice as far. The Wikipedia article on lighthouses gives the single-height formula `D = 1.22 √H`, with `H` in feet and `D` in nautical miles. That conflicts with the Wikipedia article on the horizon, where 1.22 is the constant for *statute* miles with the pure geometry, and the nautical-mile constant with feet is about 1.06 (about 1.15 with refraction, which is my own conversion of 2.084 from metres to feet). I trust the horizon article's explanation of the constants, and do not use the 1.22 figure in this course.

Putting It Together

Now the whole calculation. The code finds the luminous range from Allard's law by bisection (repeatedly halving an interval until the answer is pinned down), uses the 0.2 microlux threshold, and compares it with the geographic range. Every light intensity and height is an assumption for illustration.

```
import math

NM = 1852.0                                  # metres in a nautical mile
KM_PER_SQRT_M = 3.86                         # horizon constant, with standard refracti

def geographic_range_nm(light_m, eye_m):
    return KM_PER_SQRT_M * (math.sqrt(light_m) + math.sqrt(eye_m)) / (NM / 1000)

def transmissivity(visibility_nm):
    """Fraction of light left after 1 nm, if 5% is left after visibility_nm."""
    return 0.05 ** (1 / visibility_nm)

def luminous_range_nm(intensity_cd, visibility_nm, threshold_lx=2e-7):
    """Allard's law, E = I * T**D / (D*1852)**2, solved for D by bisection."""
    t = transmissivity(visibility_nm)
    lo, hi = 0.01, 200.0
    for _ in range(100):
        mid = (lo + hi) / 2
        e = intensity_cd * t ** mid / (mid * NM) ** 2
        if e > threshold_lx:
            lo = mid
```

```
        else:
            hi = mid
    return (lo + hi) / 2

print(round(transmissivity(10), 3))
for v in (2, 5, 10, 20):
    print(v, round(luminous_range_nm(1_000_000, v), 1))
# 0.741
# 2 6.9
# 5 14.7
# 10 25.7
# 20 44.2
```

A light of an assumed 1,000,000 cd has a luminous range of 6.9 nm in poor visibility (2 nm), 14.7 nm at 5 nm, 44.2 nm in very clear air (20 nm), and **25.7 nm at the standard 10 nm visibility, which is its nominal range**. The weather changes the answer by a factor of more than six, which is why a chart prints one agreed number and mariners are given tables to convert it.

```
for i in (10_000, 100_000, 1_000_000, 10_000_000):
    print(i, round(luminous_range_nm(i, 10), 1))
for h in (10, 30, 60, 100):
    print(h, round(geographic_range_nm(h, 5), 1))    # eye assumed 5 m up
# what the sailor actually sees: the smaller of the two
lum = luminous_range_nm(1_000_000, 10)
geo = geographic_range_nm(30, 5)
print(round(lum, 1), round(geo, 1), round(min(lum, geo), 1))
# 10000 14.3
# 100000 19.8
# 1000000 25.7
# 10000000 31.9
# 10 11.3
# 30 16.1
# 60 20.8
# 100 25.5
# 25.7 16.1 16.1
```

The first four lines show how stubborn the inverse square law is: multiplying the intensity by 100, from 100,000 cd to 10,000,000 cd, raises the nominal range only from 19.8 nm to 31.9 nm, a

factor of 1.6. The next four lines show the geographic range for an observer whose eye is 5 m above the sea: 11.3 nm for a 10 m light and 25.5 nm for a 100 m light. The final line is the point of the chapter. The 1,000,000 cd light would reach 25.7 nm in standard weather, but if it stands 30 m above the sea and the observer's eye is 5 m up, the horizon cuts it off at **16.1 nm**. Here a brighter lamp would not help. Only a taller tower, or a taller observer, would.

Nominal Range Is Not the Distance You See

A published nominal range is easy to misread. It is a statement about the light's *brightness* in standard weather, and it ignores the Earth's curvature altogether, as the Boatnews and IALA definitions both stress. A chart that says a light has a nominal range of 25 nm does not promise that a small boat will see it from 25 nm away. The boat's eye is a few metres up, and the light may be below its horizon long before its glare fades. Equally, a lamp on a very tall cliff has a long geographic range but may be too faint to see at that distance in haze. The two limits are independent, and the shorter one wins.

Common mistakes

Reading the nominal range as the distance at which the light will be seen (it is the luminous range in standard weather, and the horizon is ignored); forgetting the observer's own height in the geographic range (the two horizon distances add); expecting twice the range from twice the intensity (the inverse square law means four times); treating a height formula as exact (the refraction constants 3.57, 3.86, 2.08 and 2.10 are approximations, and the sources differ slightly); and mixing units (nautical miles, kilometres, feet and metres all appear in lighthouse sources, and the constants only work with matching units).

EXERCISE 1

A light of 250,000 cd (assumed). Find the illuminance it gives at 1, 2, 3 and 5 km, the distance at which it has fallen to 0.01 lx, and the intensity needed to give the same 0.01 lx at twice that distance. What does the answer say about doubling a lighthouse's range?

View solution

EXERCISE 2

For lights 20, 40 and 80 m above the sea, seen by an eye 3 m up, find the geographic range with and without refraction (3.86 and 3.57 km per square root of a metre), in nautical miles. Then find the height needed for a 20 nm range, and the height needed to double the range of the 40 m light.

View solution

EXERCISE 3

A light stands 25 m above the sea and is seen by an eye 4 m up. For intensities of 100,000 and 2,000,000 cd and visibilities of 2, 10 and 20 nm, find the luminous range and decide which of the luminous and geographic ranges limits what the observer sees. Then find the intensity that gives a nominal range of 15 nm.

View solution

QUICK REFERENCE

- **Inverse square law:** $E = I / d^2$; double the distance, a quarter of the light
- **Luminous range:** how far the light's strength carries it in the prevailing weather
- **Nominal range:** the luminous range when meteorological visibility is 10 nm (T = 0.74 per nm)
- **Geographic range:** set by the Earth's curve: about 3.86 (√H + √h) km, or 2.08 (√H + √h) nm, heights in metres, with standard refraction
- **Without refraction:** 3.57 km per square root of a metre; refraction adds about 8%
- **What is seen:** the smaller of the luminous and geographic ranges

COMING NEXT

How Lighthouses Work 2 is about early beacons: the Pharos of Alexandria, coal and wood fires, candles, the Eddystone towers, and Smeaton's 1759 tower and dovetailed stone.

How Lighthouses Work

Chapter 2 · Early Beacons

How Lighthouses Work 1 gave us the tools: the inverse square law, the horizon, and the difference between luminous and geographic range. This chapter uses them on the first real lights. The early story is short to tell. Builders learned to raise a fire high (the Pharos of Alexandria), then found that a fire is hungry (the coal beacon on the Isle of May), then that a candle is weak (the candle-lit Eddystone towers), and finally that a tower has to stand against the sea (John Smeaton's stone tower of 1759). Each step has a number attached, and each number can be checked.

> **WHAT WAS RUN, AND WHAT TO TRUST**
>
> Every Python snippet in this chapter and in the three exercise solutions was run, and the output in the comments is what Python printed. The historical facts (dates, heights, candle counts, coal tonnage) were read from the pages named in the text; where sources disagree, the page says so. The *physical assumptions* are mine and are labelled where they appear: about 1 cd for a candle, 24 MJ per kg for coal, 2,650 kg per cubic metre for granite, the wind pressures and friction coefficient, and the 5,000 cd fire. They show how the principle works and are not measurements of any real light. I have used the tower heights as listed in the sources, which do not include the height of the rock the tower stands on.

The Pharos of Alexandria

The Wikipedia article on the Lighthouse of Alexandria (read directly) says it was built by the Ptolemaic Kingdom: Ptolemy I Soter commissioned it shortly after declaring himself king in 305 BC, and his son Ptolemy II Philadelphus finished it, over twelve years and at a cost of "800 talents of silver". The infobox gives the construction as between 284 and 246 BC, while the article's text puts Ptolemy II's reign at 280 to 247 BC, a small mismatch that shows how loose ancient dates are. Pliny the Elder named Sostratus as the architect, but the article says this conclusion is disputed.

It had three tiers: square at the bottom, octagonal in the middle and cylindrical at the top. The **height is not settled**. The article's lead says "at least 100 metres", its infobox gives 103 to 118 m, and the medieval Arab writer Al-Idrisi (1154) reckoned about 162 m. I use 103 to 118 m below, and treat it as a range rather than a fact. The light itself is also described two ways: "a

furnace at the top", and a mirror at the apex that reflected sunlight by day with a fire lit at night. The article does not reconcile the two, and neither do I.

Its end was gradual. Earthquakes damaged it in 796 and 951, the 956 earthquake brought down the top 20 m, and the 1303 earthquake caused the final major damage. The remnant vanished in 1480 when Qaitbay built a fort on the site with the fallen stone. A medieval legend that a Byzantine agent undermined the foundations is presented by the article as legend only.

```
import math

def geographic_range_nm(light_m, eye_m):
    return 3.86 * (math.sqrt(light_m) + math.sqrt(eye_m)) / 1.852

for name, h in (("Pharos low", 103), ("Pharos high", 118), ("Pharos 'at least'", 100)):
    print(name, h, round(geographic_range_nm(h, 5), 1))    # eye assumed 5 m up
print(round(math.sqrt(118 / 103), 3))
# Pharos low 103 25.8
# Pharos high 118 27.3
# Pharos 'at least' 100 25.5
# 1.07
```

This is the geographic range formula from How Lighthouses Work 1. A sailor 5 m above the water could have seen a light at the top of the Pharos from about 25 to 27 nautical miles, if it was bright enough, and the whole disputed range of heights makes only a 7% difference to the horizon (the last line, the square root of 118/103). The height argument matters less to the range than it sounds, because range grows with the *square root* of height. Whether the fire was bright enough is the other half of the question, and for that we need a fire.

A Fire Is Hungry: The Isle of May Beacon

Open fires on towers, hills and headlands were the standard light for most of history. The best-documented Scottish one is on the Isle of May in the Firth of Forth. The Wikipedia article on the island (read directly) says a coal-fired beacon was established in 1635 (some sources say 1636), burned about **400 tons of coal a year**, needed three men to tend it, and was funded by a tonnage fee on passing ships. A search for further sources agreed on the 400 tons, and added

that up to 3 tons could be burned in a single night (this is from the summary of a Canmore entry that I did not open, so treat the 3 tons as a search summary). The beacon was replaced by Robert Stevenson's stone lighthouse, which began operating on 1 September 1816. The 150 tons a year that appears in some sources belongs to the later lighthouse, not the beacon.

```
tons_per_year = 400          # Isle of May beacon, per the sources
tons_worst_night = 3         # "up to 3 tons in a single night"
print(round(tons_per_year / 365, 2), "t per day on average")
night_hours = 12             # assumed length of the burning night
kg_per_hour = tons_worst_night * 1000 / night_hours
print(round(kg_per_hour, 1), "kg per hour in the worst night")
MJ_PER_KG = 24               # assumed heat content of coal (general knowledge)
power_mw = kg_per_hour * MJ_PER_KG / 3600
print(round(power_mw, 2), "MW of heat")
# 1.1 t per day on average
# 250.0 kg per hour in the worst night
# 1.67 MW of heat
```

On average that is 1.1 tons a day, and on the worst night about 250 kg an hour, a heat output of roughly 1.67 MW, the output of several hundred domestic electric fires (assuming about 2 kW each). Almost all of it was wasted as heat and smoke. The point is not the exact figure, which depends on the assumed 24 MJ/kg and the 12-hour night, but the scale: keeping a fire going meant hauling, storing and feeding coal every night of the year, with three people paid by shipping dues. That cost, as much as the poor light, drove the search for better lamps, which is the subject of the next chapter.

A Candle Is Weak: The Eddystone Towers

Eddystone, a reef off Plymouth, shows how the problems stack up. The Wikipedia article on Eddystone Lighthouse (read directly) lists four towers on the rock:

TOWER	FIRST LIT	BUILT OF	LIGHT	FATE
Winstanley	14 Nov 1698	wood, later clad in stone on a timber frame (about 18 m)	60 candles and a great hanging lamp	destroyed in the great storm of 1703, with Winstanley and five others

TOWER	FIRST LIT	BUILT OF	LIGHT	FATE
Rudyerd	1708	timber storeys on a base of stone courses (about 21 m)	24 candles	burned on 2 Dec 1755 after a lantern fire, after nearly 50 years
Smeaton	16 Oct 1759	granite and Portland limestone, hydraulic lime (about 22 m)	24 large tallow candles	replaced 1882; top moved to Plymouth Hoe
Douglass	18 May 1882	granite (4,668 tons) (49 m)	oil, first-order lens	still in use; automated 1982

The article's text and its infobox disagree on some details (Rudyerd's shape; Smeaton's height as 18 m in one place and 22 m in another; the year Smeaton's tower was deactivated). I use the figures from the Smeaton's Tower article, which I also read directly: 72 ft (22 m) high, 26 ft (8 m) across at the base and 17 ft (5 m) at the top. Wikipedia lists the Rudyerd candle count as 24 and Smeaton's as 24, and the fourth tower is the first where the figures are in candela (a first-order lens with two six-wick oil burners) rather than candles. In 1882 the Smeaton tower was taken down and its upper part rebuilt on Plymouth Hoe, where it stands as a memorial.

How far does a candle-lit tower shine?

The Wikipedia article on the candela says a common candle emits roughly 1 cd, and I use 1 cd per candle as an assumption. The luminous-range function is the one from How Lighthouses Work 1, repeated here so the snippet runs on its own.

```
import math

NM = 1852.0

def transmissivity(visibility_nm):
    return 0.05 ** (1 / visibility_nm)

def luminous_range_nm(intensity_cd, visibility_nm, threshold_lx=2e-7):
    t = transmissivity(visibility_nm)
    lo, hi = 0.01, 200.0
    for _ in range(100):
        mid = (lo + hi) / 2
        e = intensity_cd * t ** mid / (mid * NM) ** 2
```

```
        if e > threshold_lx:
            lo = mid
        else:
            hi = mid
    return (lo + hi) / 2

def geographic_range_nm(light_m, eye_m):
    return 3.86 * (math.sqrt(light_m) + math.sqrt(eye_m)) / 1.852

CD_PER_CANDLE = 1.0     # "roughly 1 cd" for a common candle
towers = (("Winstanley", 60, 18), ("Rudyerd", 24, 21), ("Smeaton", 24, 22))
for name, candles, h in towers:
    lum = luminous_range_nm(candles * CD_PER_CANDLE, 10)
    geo = geographic_range_nm(h, 5)
    print(name, candles, round(lum, 1), round(geo, 1), "limit:", "light" if lum < geo el
# Winstanley 60 4.7 13.5 limit: light
# Rudyerd 24 3.5 14.2 limit: light
# Smeaton 24 3.5 14.4 limit: light
```

The columns are the tower, the number of candles, the nominal range in nautical miles (luminous range at 10 nm visibility), and the geographic range for an eye 5 m up, using only the tower's height (the rock adds some more). The result is clear and a little cruel: 24 candles give a nominal range of about 3.5 nm, while the horizon would allow over 14 nm. **The light, not the tower, was the limit**, and 60 candles reach only 4.7 nm, not 2.5 times as far, because of the inverse square law. Treat this as an order-of-magnitude picture: a lantern's candles are spread around the compass, not all aimed at one sailor, and the real range depends on the glass, the weather and the watcher. Even so, the gap between 3.5 and 14 nm explains the next three chapters: more light on the same tower.

The first two towers were largely of wood, and neither survived the sea or the fire. John Smeaton's third tower (work began in 1756; the foundation stone, weighing 2¼ tons, was carried out to the reef on 12 June 1756, and the light was first lit on 16 October 1759) was the first to be thought through as engineering. The Smeaton's Tower article (read directly) says it was built of granite blocks joined with **dovetail joints and marble dowels**, used **hydraulic lime** (a Roman-era cement that sets underwater, which Smeaton rediscovered), and was **modelled on**

an oak tree, wide at the base and narrowing upwards, for strength. The cost was £40,000. The article on hydraulic lime (also read directly) says hydraulic limes set by reacting with water, so they harden even when submerged, unlike air lime, which hardens by absorbing carbon dioxide. It credits Smeaton with pioneering hydraulic lime in the 18th century.

Why the base is wider

A tower of stone has to carry its own weight. The lower courses carry everything above them, so the stress there is greatest. A taper puts more stone where the load is greatest and saves it where it is not. The code below models Smeaton's tower as a solid truncated cone with the sourced dimensions (22 m high, 8 m across at the base, 5 m at the top) and an assumed granite density of 2,650 kg per cubic metre. The real tower is not solid (it has rooms and a stair), so these figures overestimate the weight and the stress. They show the pattern, not the real loads.

```
import math

# Smeaton's tower as a solid truncated cone (an overestimate: the real tower has rooms)
H = 22.0                 # m, height
D_BASE, D_TOP = 8.0, 5.0 # m, diameters
RHO = 2650.0             # kg/m3, assumed density of granite (general knowledge)
G = 9.81

def diameter_at(y):
    """Diameter at height y metres above the base."""
    return D_BASE + (D_TOP - D_BASE) * y / H

def area(d):
    return math.pi * d ** 2 / 4

def weight_above(y, steps=2000):
    """Weight (N) of the cone slice from height y up to the top."""
    dy = (H - y) / steps
    total = 0.0
    for i in range(steps):
        total += area(diameter_at(y + (i + 0.5) * dy)) * dy
    return total * RHO * G

for y in (0, 5, 10, 15, 20):
    w = weight_above(y)
    stress = w / area(diameter_at(y))
    print(y, round(diameter_at(y), 2), round(w / 1e6, 2), "MN", round(stress / 1e3), "kP
print(round(weight_above(0) / G / 1000), "tonnes in total")
```

```
# 0 8.0 19.32 MN 384 kPa
# 5 7.32 13.32 MN 317 kPa
# 10 6.64 8.35 MN 241 kPa
# 15 5.95 4.3 MN 154 kPa
# 20 5.27 1.08 MN 49 kPa
# 1969 tonnes in total
```

The columns are the height above the base (m), the diameter there (m), the weight of everything above (meganewtons) and the average compressive stress (kilopascals). The load climbs from 1.08 MN at 20 m to 19.32 MN at the base, and the stress from 49 kPa to 384 kPa, so the lowest stones bear nearly eight times what the highest do. (A kilopascal is a thousand newtons per square metre. Granite is generally far stronger than this in compression (general knowledge, not checked here), so the stone itself is not near failing under its own weight. The sea is the threat, as How Lighthouses Work 6 will show.) The solid model gives about 1,969 tonnes. The real tower, which is hollow in part, must weigh less; I have not verified its true mass.

Does the wind tip it over?

A second check is whether a gust could overturn the tower. A force at half the height (a crude assumption) tries to tip the tower about the edge of its base, while the weight acts through the centre, a base radius away. The ratio of the two moments is a safety factor. The wind pressures here are assumed round numbers and the 0.6 is an assumed drag factor for a round shape.

```
import math

H, D_BASE, D_TOP = 22.0, 8.0, 5.0
RHO, G = 2650.0, 9.81

# weight of the cone (frustum)
r1, r2 = D_BASE / 2, D_TOP / 2
volume = math.pi * H * (r1**2 + r1*r2 + r2**2) / 3
weight = volume * RHO * G

# wind: illustrative pressure on the projected area (a trapezoid, mean width)
for pressure in (1000, 3000, 6000):        # Pa, assumed gusts
    mean_width = (D_BASE + D_TOP) / 2
    area_proj = mean_width * H
    force = pressure * area_proj * 0.6      # 0.6: assumed drag factor for a round shape
    arm = H / 2                             # crude: force acts at half height
```

```
    overturn = force * arm
    # the tower tips about the edge of the base: restoring arm = base radius
    restore = weight * r1
    print(pressure, round(force / 1e3), "kN", round(overturn / 1e6, 2), round(restore /
# 1000 86 kN 0.94 77.3 81.9
# 3000 257 kN 2.83 77.3 27.3
# 6000 515 kN 5.66 77.3 13.6
```

The columns are the wind pressure (Pa), the force (kN), the overturning moment (MN·m), the restoring moment (MN·m) and the safety factor. Even at 6,000 Pa, a violent gust, the weight resists with a factor of 13.6. Wind alone is not what kills a squat stone tower; the sea is, which is why the shape and joints matter more than the weight. Waves hit a reef tower far harder than air, and also pull at individual stones, and that is where dovetails come in.

What a dovetail does

A dovetail is a joint cut so that the two pieces cannot be pulled apart in the direction of the load. In a course of masonry, a wave can push one stone out of its place, held back only by friction. If the stones are interlocked, the wave must move the whole ring at once. A toy model shows the idea: friction on each stone is the friction coefficient times the weight on it, and a ring of *n* interlocked stones resists *n* times as much. Every number below is illustrative, and this is a model of the principle, not a calculation of Smeaton's joints.

```
# Toy model: a wave pushes sideways on ONE block in a ring of stones.
G = 9.81
MU = 0.6                    # assumed friction coefficient, stone on stone
block_t = 2.0               # assumed block mass, tonnes (illustrative)
load_t = 4.0                # assumed mass of masonry resting on each block, tonnes

def sliding_resistance_kn(n_blocks_engaged):
    """Friction resistance if n blocks must move together."""
    return MU * (block_t + load_t) * 1000 * G * n_blocks_engaged / 1000

alone = sliding_resistance_kn(1)
for n in (1, 4, 8, 12):
    r = sliding_resistance_kn(n)
    print(n, round(r), "kN", round(r / alone, 1), "x")
# 1 35 kN 1.0 x
# 4 141 kN 4.0 x
```

```
# 8 283 kN 8.0 x
# 12 424 kN 12.0 x
```

One block resists 35 kN by friction alone, and a locked ring of 12 resists 424 kN, twelve times as much. The marble dowels and the hydraulic lime add to this, but the numbers say nothing about the real strength of Smeaton's joints, which I did not find a source for. What the model does show is why the idea works: it makes a hundred small stones act as one big one.

Common mistakes

Quoting one height for the Pharos as if it were settled (the sources range from "at least 100 m" to 162 m, and the light itself is described two ways); mixing up the Eddystone towers (Winstanley 1698 and Rudyerd 1708 were destroyed, Smeaton's of 1759 was replaced in 1882, and Douglass's of 1882 is the one in use); treating "a candle is about 1 cd" as exact (it is a rough figure and a lantern's candles are not all pointed at you); forgetting the rock's height when computing a geographic range (the sources give tower heights only); and treating the solid-cone weight as the real weight (Smeaton's tower is not solid, so the figures are upper bounds).

EXERCISE 1

A beacon burns 250 kg of coal an hour. Winters have 180 nights of 14 hours lit, summers 185 nights of 8 hours. Find the tonnes burned a year, compare with the 400 tons a year of the Isle of May beacon, find the burn rate that 400 tons over 365 nights of 12 hours implies, and the heat power and yearly heat energy (assume 24 MJ per kg).

View solution

EXERCISE 2

Find the luminous range of 24 candles (about 1 cd each) at visibilities of 5, 10 and 20 nm, the number of candles for a 10 nm nominal range, and the ratio of the nominal ranges of Winstanley's 60 candles and Smeaton's 24. Then, for a coal fire of an assumed 5,000 cd on a 103 m tower seen from an eye 5 m up, decide whether the light or the horizon limits the range.

View solution

EXERCISE 3

An imagined solid granite tower, 30 m high, 10 m across at the base and 6 m at the top. Find the average compressive stress at heights 0, 10 and 20 m, compare its weight and base stress with a plain 10 m cylinder of the same height, and use the toy dovetail model (friction 0.5, 3 t blocks with 5 t resting on each) to compare one block with a ring of ten.

View solution

QUICK REFERENCE

- **Pharos of Alexandria:** built under Ptolemy I and II, third century BC; height 103 to 118 m by one count, other sources differ; brought down by earthquakes between 796 and 1303
- **Isle of May beacon:** coal fire from 1635 or 1636, about 400 tons a year, three men; replaced by Robert Stevenson's lighthouse in 1816
- **Eddystone:** Winstanley (1698, storm 1703), Rudyerd (1708, fire 1755), Smeaton (1759), Douglass (1882)
- **Smeaton:** dovetailed granite, marble dowels, hydraulic lime, an oak-tree taper; 22 m high, 8 m across at the base, 5 m at the top
- **Candle light:** about 1 cd each, so 24 candles gives a nominal range of about 3.5 nm, far below the horizon
- **Taper:** the lower stones bear all the weight above them, so the base needs the most stone

COMING NEXT

How Lighthouses Work 3 is about lamps and fuel: open flames, the Argand lamp, whale and colza oil, the paraffin vapour burner, the electric arc, and modern bulbs and LEDs.

How Lighthouses Work

Chapter 3 · Lamps and Fuel

How Lighthouses Work 2 ended on a gap: 24 candles on a tower whose horizon was over 14 nautical miles shone only about 3.5 nautical miles. The tower was good enough and the light was not. This chapter follows the light itself, from the open flame to the Argand lamp, through the oils and the paraffin burner, to the electric arc, the filament bulb and the LED. Each step is a bigger number of candela, or a smaller bill for fuel, or a longer-lived lamp, and each can be put through the range calculation from How Lighthouses Work 1.

WHAT WAS RUN, AND WHAT TO TRUST

Every Python snippet in this chapter and in the three exercise solutions was run, and the output in the comments is what Python printed. The historical facts (dates, outputs, efficacies, lifetimes) were taken from the Wikipedia pages named in the text, as fetched; where those pages disagree with each other, the chapter says so. The *physical assumptions* are mine and are labelled where they appear: 1 cd for a candle, 4π lumens per candela (a flame shining evenly in every direction), 0.2 lm/W for a wick lamp, 37 MJ per kg for lamp oil, a 12-hour night, the 1,000 cd and 2,000 cd starting lights, the 1,500 lm lamp, and the light heights of 20, 40 and 60 m. They show how the principle works and are not measurements of any real light. The page for Bell Rock did not say which lamps were lit in 1811, so I make no claim about them.

The Problem: A Flame Is a Dim, Hungry Light

A flame makes light from burning fuel, and the numbers are poor. The Wikipedia article on luminous efficacy says a candle gives about 0.1 to 0.2 lumens for every watt of heat it burns, and a 2800 K tungsten filament about 15. A lumen is the light flowing out of a source, a candela is how much of it goes in one direction, and a source shining evenly in all directions gives 4π (12.57) lumens per candela. The Wikipedia article on the kerosene lamp uses the same conversion, 12.57 lumens per candlepower. That is where this chapter's first assumption comes from.

A better lamp therefore does one of two things: it packs more light into one flame, so that a lighthouse needs fewer burners, or it burns fuel more completely, so that the nightly bill falls. The history below is a series of attempts at one or the other.

The Argand Lamp: A Flame With Its Own Chimney

The Argand lamp was invented by Aimé Argand. The Wikipedia article on the lamp gives **1780**; the article on lighthouses gives **1782**, and says the lamp was first produced by Matthew Boulton with Argand in 1784. Those sources disagree, so I give the date as 1780 to 1782 and rely on neither. The design is the same in both: a wick shaped like a sleeve, so that air passes through the middle as well as around the outside, and a glass chimney that steadies the flame and improves the airflow. Fuel runs by gravity from a reservoir above the burner. The lighthouse article says it "became the standard for lighthouses for over a century."

The lamp article gives the output as **6 to 10 candela**, and quotes Thomas Jefferson calling it "a light equal to six or eight candles". The fuels named are whale oil, seal oil, colza, olive oil or another vegetable oil. Colza is rapeseed oil: the article on rapeseed oil says it was used with the Argand burner because it was cheaper than whale oil. The sources I read give no prices, so I do not put a number on "cheaper". What does the better burner do for a lantern?

```
import math

NM = 1852.0

def transmissivity(visibility_nm):
    return 0.05 ** (1 / visibility_nm)

def luminous_range_nm(intensity_cd, visibility_nm, threshold_lx=2e-7):
    t = transmissivity(visibility_nm)
    lo, hi = 0.01, 200.0
    for _ in range(100):
        mid = (lo + hi) / 2
        e = intensity_cd * t ** mid / (mid * NM) ** 2
        if e > threshold_lx:
            lo = mid
        else:
            hi = mid
    return (lo + hi) / 2

def geographic_range_nm(light_m, eye_m):
    return 3.86 * (math.sqrt(light_m) + math.sqrt(eye_m)) / 1.852

# Argand lamp: 6 to 10 cd per the Wikipedia article; a candle is about 1 cd
sources = (("One candle", 1), ("Argand, low", 6), ("Argand, high", 10), ("24 candles", 24
```

```
for name, cd in sources:
    print(name, cd, round(luminous_range_nm(cd, 10), 1))
print(round(luminous_range_nm(10, 10) / luminous_range_nm(1, 10), 2))
# One candle 1 1.0
# Argand, low 6 2.1
# Argand, high 10 2.6
# 24 candles 24 3.5
# 2.5
```

The columns are the source, its intensity in candela and its nominal range in nautical miles (10 nm visibility); the last line is the range of the 10 cd lamp divided by that of one candle. One Argand burner of 10 cd reaches about 2.6 nm against 1.0 nm for one candle: ten times the intensity gives only about 2.5 times the range, the inverse square law at work again, plus the loss in the air. A lantern that wants the 3.5 nm of 24 candles needs four Argands of 6 cd or three of 10 cd, in place of two dozen candles.

What does a lamp cost to feed?

Fuel use can be estimated from the efficacy and the energy in the oil. Both are assumptions, not measurements. In particular, I have assumed that a wick lamp has the top of the candle range, 0.2 lm/W, which is a guess for an Argand lamp: its steadier flame may do better. The 37 MJ per kg for oil is general knowledge, not read from a source. Treat the result as an order of magnitude.

```
import math

LM_PER_CD = 4 * math.pi      # lumens per candela if the flame shone evenly in all direc
EFFICACY = 0.2               # lm/W, assumed: the top of the 0.1 to 0.2 range given for
MJ_PER_KG = 37.0             # assumed energy content of lamp oil (general knowledge)
NIGHT_H = 12.0               # assumed burning night

for name, cd in (("One candle", 1), ("Argand, 8 cd", 8), ("Argand, 10 cd", 10)):
    lumens = cd * LM_PER_CD
    watts = lumens / EFFICACY
    mj_night = watts * NIGHT_H * 3600 / 1e6
    kg_night = mj_night / MJ_PER_KG
    print(name, round(lumens, 1), "lm", round(watts), "W", round(kg_night, 2), "kg per n
# One candle 12.6 lm 63 W 0.07 kg per night
# Argand, 8 cd 100.5 lm 503 W 0.59 kg per night
```

```
# Argand, 10 cd 125.7 lm 628 W 0.73 kg per night
```

The columns are the lamp, its lumens, the heat power in watts, and the oil burned in a 12-hour night. On these assumptions a single 8 cd Argand burns about 0.6 kg of oil a night and is a heat source of about 500 W, against 63 W for a candle. Since the model uses the same efficacy for both, the fuel simply follows the light: one Argand does the work of eight candles for about eight times the fuel (0.59 against 0.07 kg). In this model a better burner saves wicks and tending, not oil. If a real Argand is more efficient than a candle, as it may well be, the model understates the saving; I have no source for the true figure.

Oils and Paraffin

The Argand burner is a burner, and the fuel is a separate question. Whale oil, seal oil, colza and olive oil all burned in it (lighthouse article). Kerosene, also called paraffin, is the fuel that followed. The Wikipedia article on the kerosene lamp says Abraham Gesner invented a substitute for whale oil in 1846, distilled from coal, and that Ignacy Łukasiewicz built the most modern and popular versions of the paraffin lamp in Lemberg in 1853. The lighthouse article says kerosene became popular in the 1870s, and that electricity and acetylene gas began replacing it "around the turn of the 20th century". The kerosene lamp article also says a centre-draft round-wick lamp gives "three to four times the output" of a flat-wick lamp, which is the Argand idea once more.

The big jump for lighthouses was the **paraffin vapour burner**. According to the lighthouse article it was invented in 1901 by Arthur Kitson and improved by David Hood at Trinity House, and gave "an output of over six times the luminosity of traditional oil lights". The article does not describe how the burner works, so I do not either. An 85 mm Chance Brothers unit made around 1914 for Sumburgh Head was used until 1976. How much does six times the luminosity buy?

```
import math

NM = 1852.0

def transmissivity(visibility_nm):
```

```
    return 0.05 ** (1 / visibility_nm)

def luminous_range_nm(intensity_cd, visibility_nm, threshold_lx=2e-7):
    t = transmissivity(visibility_nm)
    lo, hi = 0.01, 200.0
    for _ in range(100):
        mid = (lo + hi) / 2
        e = intensity_cd * t ** mid / (mid * NM) ** 2
        if e > threshold_lx:
            lo = mid
        else:
            hi = mid
    return (lo + hi) / 2

def geographic_range_nm(light_m, eye_m):
    return 3.86 * (math.sqrt(light_m) + math.sqrt(eye_m)) / 1.852

# Illustrative: a light of BASE_CD, then lights that are 2x, 6x, 10x and 100x brighter
BASE_CD = 1000.0       # assumed starting intensity
base = luminous_range_nm(BASE_CD, 10)
for factor in (1, 2, 6, 10, 100):
    r = luminous_range_nm(BASE_CD * factor, 10)
    print(factor, round(r, 1), "nm", round(r / base, 2), "x the range")
# 1 9.4 nm 1.0 x the range
# 2 10.8 nm 1.15 x the range
# 6 13.1 nm 1.4 x the range
# 10 14.3 nm 1.52 x the range
# 100 19.8 nm 2.11 x the range
```

The columns are the intensity factor, the nominal range and how many times the starting range that is. The starting light of 1,000 cd is an assumption. The result is the point of the whole chapter: a light *six* times as bright reaches only **1.4** times as far, and a light a *hundred* times as bright only about 2.1 times. A claim of "six times the luminosity" is a claim about candela, not about miles. Adding lamps has steeply diminishing returns, which is why How Lighthouses Work 4 turns to lenses, which gather the light that a lamp already makes and send it where the sailor is.

The Electric Arc

The carbon arc lamp makes light from an electric spark between two carbon rods. The Wikipedia article on arc lamps says Humphry Davy demonstrated it in the early 1800s, with sources citing 1802, 1805, 1807 or 1809, so the date is not settled. Arc lamps need an electrical supply, which for a lighthouse in the 1800s meant its own generator (the lighthouse articles below mention steam-driven ones).

The lighthouse sources disagree about the first use, and I list the claims side by side rather than choose between them:

EVENT	THE SOUTH FORELAND ARTICLE	THE LIGHTHOUSE ARTICLE	THE SOUTER ARTICLE
First trial	December 1858 to early 1860 at South Foreland Upper Light, "the first lighthouse to use electric light"	says South Foreland was "the first tower to successfully use an electric light" in 1875	says earlier electric lights were installed at Dungeness (1862)
Permanent installation	1872 at South Foreland, with four steam-driven Holmes generators; 1875 is the date of an illustration, not of the installation	not given	not given
Built for electricity	Souter, 1871	Souter, 1871	11 January 1871; "the first lighthouse in the world to be actually designed and built" for electricity
Equipment	Frederick Hale Holmes's generators; Faraday as Trinity House's adviser	carbon arc lamps, steam-driven magneto	two Holmes generators on two steam engines; James Douglass the designer

The 1875 date in the lighthouse article may be the date of the illustration that the South Foreland article mentions, but I have not checked that. The 1858 trial, the 1862 Dungeness installation and the 1871 Souter lighting come from pages that agree with each other, so those are the ones I would quote. The Souter article gives the light as a carbon arc of "800,000 candle power", visible up to 26 miles (it does not say statute or nautical). That intensity is the source's claim; I have not found how it was measured, and it may include the effect of the lens. What range does it predict?

```
import math

NM = 1852.0

def transmissivity(visibility_nm):
    return 0.05 ** (1 / visibility_nm)

def luminous_range_nm(intensity_cd, visibility_nm, threshold_lx=2e-7):
    t = transmissivity(visibility_nm)
    lo, hi = 0.01, 200.0
    for _ in range(100):
        mid = (lo + hi) / 2
        e = intensity_cd * t ** mid / (mid * NM) ** 2
        if e > threshold_lx:
            lo = mid
        else:
            hi = mid
    return (lo + hi) / 2

def geographic_range_nm(light_m, eye_m):
    return 3.86 * (math.sqrt(light_m) + math.sqrt(eye_m)) / 1.852

# Souter, per the Wikipedia article: carbon arc, "800,000 candle power", visible up to 2
ARC_CD = 800_000
for vis in (10, 20):
    print("visibility", vis, "nm: luminous range", round(luminous_range_nm(ARC_CD, vis),
for light_m in (20, 40, 60):       # assumed light heights; the sources used here give n
    geo = geographic_range_nm(light_m, 5)
    print("height", light_m, "m: geographic range", round(geo, 1), "nm")
# visibility 10 nm: luminous range 25.1 nm
# visibility 20 nm: luminous range 43.0 nm
# height 20 m: geographic range 14.0 nm
# height 40 m: geographic range 17.8 nm
# height 60 m: geographic range 20.8 nm
```

The first two lines are the luminous range at 10 and 20 nm visibility; the last three are the geographic range for three assumed light heights, with an eye 5 m up. At 10 nm visibility an 800,000 cd light has a luminous range of 25.1 nm, close to the 26 miles in the article, which is reassuring but not a proof, as the units are uncertain. The real lesson is the second half. A light this bright is limited by the **horizon**, not by the light: for a tower of 20 to 60 m (assumed) the

geographic range is only 14.0 to 20.8 nm, however clear the night. Beyond a point, more candela does nothing and the next improvement must come from height. The candles of How Lighthouses Work 2 were limited by the light; the arc is limited by the earth.

Bulbs and LEDs

Filament bulbs followed. The Wikipedia article on the incandescent light bulb gives Swan's working demonstration on 17 January 1879 and Edison's first successful test on 22 October 1879. It gives typical efficacies of 16 lm/W for a 120 V bulb and 24 lm/W for quartz halogen, and a theoretical maximum of 52 lm/W for a tungsten filament. The luminous efficacy article gives about 15 lm/W for a typical 2800 K tungsten bulb, 17 to 20 for halogen, and about 102 lm/W for LED screw-base lamps (up to 172 for some tubes). The lighthouse article gives a filament lifetime of "about 1,000 hours" and an LED lifetime of "50,000 to 100,000 hours", says LEDs were widely used by 2020, and that a typical LED system costs about 20,000 euros according to a supplier. Here is the comparison laid out:

SOURCE	EFFICACY (LM/W)	LIFETIME (HOURS)	WHERE IT COMES FROM
Candle	0.1 to 0.2	not applicable	luminous efficacy article
Tungsten filament	about 15	about 1,000	luminous efficacy and lighthouse articles
Halogen	17 to 20 (24 for quartz in one source)	1,000 to 2,000 for general lamps	luminous efficacy and bulb articles
White LED	about 102 (some tubes 172)	50,000 to 100,000	luminous efficacy and lighthouse articles

The halogen figures differ slightly between the two articles (17 to 20 against 16 or 24); I use 20 below. The bulb article's 1,000 to 2,000 hours is for general-service lamps, so the 1,000 hours I use is the cautious end, and I have applied the filament lifetime to halogen as well, which is an assumption. Now put those numbers in a lamp of 1,500 lumens running every night:

```
# Efficacy and lifetime figures from the Wikipedia articles on luminous efficacy and lig
SOURCES = (                       # name, lm/W, lifetime in hours
    ("Incandescent filament", 15, 1000),
    ("Halogen (top of 17 to 20)", 20, 1000),     # the 1,000 h lifetime is the filament-
```

```
    ("White LED, screw-base", 102, 50000),       # the low end of the 50,000 to 100,000 h
)
NEED_LM = 1500.0                # assumed light output wanted from the lamp
HOURS_PER_YEAR = 4380           # assumed: 12 hours of darkness, every night

for name, eff, life in SOURCES:
    watts = NEED_LM / eff
    kwh = watts * HOURS_PER_YEAR / 1000
    changes = HOURS_PER_YEAR / life
    print(name, round(watts), "W", round(kwh), "kWh a year", round(changes, 2), "lamp ch
# Incandescent filament 100 W 438 kWh a year 4.38 lamp changes a year
# Halogen (top of 17 to 20) 75 W 328 kWh a year 4.38 lamp changes a year
# White LED, screw-base 15 W 64 kWh a year 0.09 lamp changes a year
```

The columns are the source, the power it draws, the electricity used in a year and the number of lamp changes a year. For the same 1,500 lumens the filament draws 100 W and the LED about 15 W, and the energy falls from 438 to 64 kWh a year. The bigger change for a lighthouse is the last column: 4.4 filament changes a year against one LED change in about eleven years (0.09 a year). A light that needs a bulb changed every few months needs someone nearby; one that runs for years does not. That is one reason the unmanned light became practical, a story that How Lighthouses Work 8 tells.

Common mistakes

Treating the Argand lamp's date as settled (1780 and 1782 both appear in the sources); reading "six times the luminosity" as "six times the range" (the range grew by about 1.4 times in the model); calling 1875 the first electric light at South Foreland (the South Foreland article gives an 1858 trial and a 1872 permanent installation, and says 1875 is only an illustration); quoting "800,000 candle power" as a measured fact when it is the source's figure and the units of the range are unclear; and treating the 0.2 lm/W and 37 MJ per kg of the fuel estimate as data (they are labelled assumptions).

EXERCISE 1

How many Argand lamps of 6 cd, and how many of 10 cd, match the intensity of 24 candles? Find the nominal range (10 nm visibility) of three 8 cd Argands treated as one 24 cd source and of 24 candles. Then estimate the oil one 8 cd Argand burns in a 12-hour night and in 365 nights, and compare the mass with the 400 tons of coal the Isle of May beacon burned in a year.

View solution

EXERCISE 2

Take an assumed 2,000 cd light and a lamp six times as bright. Find both nominal ranges, and the ratio between them, at visibilities of 5, 10 and 20 nm. Then use a bisection search to find the factor by which the intensity must be multiplied to double the nominal range at 10 nm visibility, and compare with the factor that the inverse square law alone would give.

View solution

EXERCISE 3

A lamp must give 1,500 lm for 4,380 hours a year. Compare a 15 lm/W filament of 1,000 hours life with a 102 lm/W LED of 50,000 hours life: the power, the kWh a year and the number of lamps needed in 10 years. Then compare the nominal ranges of the Souter arc (800,000 cd) and the Bell Rock's current 1,900,000 cd, and decide whether the listed 18 nmi range of the Bell Rock agrees with the model.

View solution

QUICK REFERENCE

- **Argand lamp:** hollow wick and chimney; 1780 or 1782 by two sources; 6 to 10 cd; whale oil, colza or other oils; standard for lighthouses for over a century
- **Colza:** rapeseed oil, cheaper than whale oil in the Argand burner (no prices found)
- **Paraffin (kerosene):** Gesner 1846, Łukasiewicz 1853, popular in the 1870s; vapour burner 1901 (Kitson, Hood), over six times the luminosity of oil lights
- **Electric arc:** South Foreland trial 1858, Dungeness 1862, Souter lit 11 January 1871, "800,000 candle power" (the source's figure)
- **Filament and LED:** about 15 lm/W and 1,000 hours against about 102 lm/W and 50,000 to 100,000 hours
- **Range:** six times the intensity gave about 1.4 times the range in the model; for a very bright light the horizon, not the lamp, sets the limit

COMING NEXT

How Lighthouses Work 4 is about lenses: reflectors against lenses, Buffon and Fresnel, the Fresnel lens principle, lens orders, prisms, and why a stepped lens is lighter.

How Lighthouses Work

Chapter 4 · Lenses

How Lighthouses Work 3 ended on a limit: a lamp six times as bright reached only about 1.4 times as far, so more lamps are a poor way to make a light reach further. The better way is to stop wasting the light a lamp already makes. A flame sends light in every direction and only a thin slice of it travels toward a ship. This chapter is about the glass and metal that bend the rest into that slice: first the reflector, then the lens, then the stepped lens of Augustin-Jean Fresnel that put an end to the reflectors.

WHAT WAS RUN, AND WHAT TO TRUST

Every Python snippet in this chapter and in the three exercise solutions was run, and the output in the comments is what Python printed. The historical facts (dates, shares of light, focal lengths, weights) were taken from the Wikipedia pages on Fresnel lenses and on lighthouses, as fetched; where the pages disagree, the chapter says so. The *physical assumptions* are mine and are labelled where they appear: a 1,000 cd starting intensity, a 10 cd burner, a flame 2 cm across (10 mm radius), a glass refractive index of 1.5, a glass density of 2,500 kg per cubic metre, a stepped panel 20 mm thick, and a solid lens edge of 5 mm. The lens is modelled as a simple round lens with the flame at its focus. That shows how focal length and flame size set the beam, but it is not a design for a real lens, and the candela it prints are not measurements of any real light. The statements about how refraction works (it happens at the surfaces) are general optics, not read from a source.

Reflectors: Catching the Light That Goes the Wrong Way

A bare flame lights the sky and the sea equally and the sky does not need it. The first practical fix was a mirror. The lighthouse article says the Argand hollow-wick lamp and the parabolic reflector were introduced in the late 18th century, and a steady flame made focusing practical in the first place. It also says Hutchinson's 1777 catoptric system was the first practical optical system, focusing light into a concentrated beam. A parabolic mirror sends the light from a small flame at its focus out as a parallel beam. That is general geometry, not a claim from a source.

But a reflector wastes most of the light. The two Wikipedia articles give figures that do not quite line up:

CLAIM	SOURCE	SHARE OF THE LAMP'S LIGHT
Parabolic reflectors of the time	the lighthouse article (citing an NPS source)	20% focused
Fresnel's reflectors	the Fresnel lens article	"only about half" of the incident light reflected
A Fresnel lens	the lighthouse article	85% focused

The 20% and the "about half" are not quite the same quantity (focused into the beam, against reflected at all), so I treat them as a source conflict that I cannot resolve, and I have not found how either was measured. The lighthouse article also says the first Fresnel lens at Cordouan "increased the luminosity of the lighthouse lamp by a factor of four". The 85% and the 20% differ by 4.25 times. What does that do to the range?

```
import math

NM = 1852.0

def transmissivity(visibility_nm):
    return 0.05 ** (1 / visibility_nm)

def luminous_range_nm(intensity_cd, visibility_nm, threshold_lx=2e-7):
    t = transmissivity(visibility_nm)
    lo, hi = 0.01, 200.0
    for _ in range(100):
        mid = (lo + hi) / 2
        e = intensity_cd * t ** mid / (mid * NM) ** 2
        if e > threshold_lx:
            lo = mid
        else:
            hi = mid
    return (lo + hi) / 2

def geographic_range_nm(light_m, eye_m):
    return 3.86 * (math.sqrt(light_m) + math.sqrt(eye_m)) / 1.852

# Wikipedia (lighthouse article): a Fresnel lens focused 85% of a lamp's light, paraboli
# Wikipedia (Fresnel lens article): Fresnel's reflectors reflected "only about half" of
BASE_CD = 1000.0       # assumed: the intensity of the 20% reflector
CLAIMS = (("Reflector, 20% claim", 0.20), ("Reflector, half", 0.50), ("Fresnel lens, 85%
```

```
base = luminous_range_nm(BASE_CD, 10)
for name, share in CLAIMS:
    cd = BASE_CD * share / 0.20
    r = luminous_range_nm(cd, 10)
    print(name, share, round(cd), "cd", round(r, 1), "nm", round(r / base, 2), "x the ra
print(round(0.85 / 0.20, 2))
# Reflector, 20% claim 0.2 1000 cd 9.4 nm 1.0 x the range
# Reflector, half 0.5 2500 cd 11.2 nm 1.2 x the range
# Fresnel lens, 85% claim 0.85 4250 cd 12.4 nm 1.32 x the range
# 4.25
```

The columns are the source, its share of the light, its intensity in candela (the 1,000 cd for the 20% reflector is an assumption), its nominal range in nautical miles at 10 nm visibility, and the range as a multiple of the reflector's. The last line is 0.85 divided by 0.20. The lens at 4,250 cd has 4.25 times the intensity but only 1.32 times the range, which is the inverse square law and the loss in the air, as in How Lighthouses Work 3. Even so, a third more range is worth having, and the lens did it with the same lamp.

From Buffon to Fresnel

The idea of the stepped lens is older than Fresnel. A lens bends light only at its two surfaces; the glass between them does no bending, adds weight and absorbs light. That is general optics, not a quote. A stepped lens keeps the surfaces and removes the glass in the middle. The timeline from the Fresnel lens article:

DATE	WHO	WHAT
1748	Buffon	replaced a convex lens with concentric annular prisms ground as steps in one piece of glass, to reduce weight and absorption
1790 (see below)	Condorcet	suggested making the annular sections separately and mounting them in a frame
1788 and 1789	Thomas Rogers	proposed a lighthouse lens to Trinity House in 1788; lenses installed at Portland Bill in 1789, but "much of the light was wasted by absorption in the glass"
1811 and 1820	Brewster	proposed a Condorcet-like system in 1811; advocating it for British lighthouses by 1820

DATE	WHO	WHAT
June 1819	Fresnel	seconded to the French Commission des Phares (set up in 1811); recommended stepped lenses (exact date unknown)
March 1820, 13 April 1821	Fresnel	prototype finished; a full eight-panel version demonstrated
25 July 1823	Fresnel	the first lighthouse Fresnel lens lit at Cordouan, visible more than 32 km (20 mi) out

A source conflict: the article dates Condorcet's proposal to 1790 but says secondary sources give 1773 or 1788, so I do not rely on any of them. The article also says Fresnel died on 14 July 1827, aged 39, having designed the catadioptric prism but not seen a full-sized version. The lighthouse article gives the same Cordouan year (1823) and the same visibility, more than 20 miles (32 km). Buffon, Condorcet, Rogers and Brewster all had the idea; Fresnel is the one who made it work.

Lenses, Prisms and Why They Are Lighter

A lighthouse lens is built from rings. A ring near the middle *refracts*: the light goes in one side, bends and comes out as a beam. The article calls these **dioptric** parts: refraction only. For light leaving the flame at a steep angle, the article describes **catadioptric** prisms: each does two refractions and one total internal reflection, which it says captures more oblique light and avoids the loss from a silvered mirror. The light that a reflector wastes sideways is gathered into the beam.

The size of a lens is given by its **order**. According to the lighthouse article, a first-order lens is the largest, most powerful and most expensive of the usual sizes, and coastal lighthouses generally use first, second or third order. The Fresnel lens article lists these focal lengths. The numbering differs between its "modern" and "Fresnel" orders, so I use the modern ones.

ORDER (MODERN)	FOCAL LENGTH (MM)	FIRST INSTALLED
Hyper-radial	1330	1887
Mesoradial	1125	1909
1st	920	1823

ORDER (MODERN)	FOCAL LENGTH (MM)	FIRST INSTALLED
2nd	700 to 750	not given
3rd	500	1825
4th	250	not given
6th	150	not given

The article lists more orders than shown, down to an 8th order of 70 to 75 mm focal length; I have kept the ones used below. It gives one weight: a first-order assembly about 2.54 m tall weighing about 1.5 tonnes (3,300 lb). Why does focal length matter? The flame is not a point. A flame of radius r at a focal distance f spreads the beam by a half-angle whose tangent is r divided by f, so a long focal length gives a narrow beam, and a narrow beam is a bright one. Here is that model for one 10 cd burner and a 10 mm flame radius (both assumed), with 85% of the light collected (the source's share for a lens), at the focal lengths of six orders (700 to 750 mm shown as 725, my midpoint):

```
import math

NM = 1852.0

def transmissivity(visibility_nm):
    return 0.05 ** (1 / visibility_nm)

def luminous_range_nm(intensity_cd, visibility_nm, threshold_lx=2e-7):
    t = transmissivity(visibility_nm)
    lo, hi = 0.01, 200.0
    for _ in range(100):
        mid = (lo + hi) / 2
        e = intensity_cd * t ** mid / (mid * NM) ** 2
        if e > threshold_lx:
            lo = mid
        else:
            hi = mid
    return (lo + hi) / 2

def geographic_range_nm(light_m, eye_m):
    return 3.86 * (math.sqrt(light_m) + math.sqrt(eye_m)) / 1.852

FLAME_RADIUS_M = 0.01                # assumed: a flame about 2 cm across
```

```
LUMENS = 10 * 4 * math.pi            # assumed: one 10 cd burner shining evenly in all
CAPTURE = 0.85                       # the lens share from the lighthouse article
ORDERS = (("Hyper-radial", 1330), ("1st", 920), ("2nd", 725), ("3rd", 500), ("4th", 250)

for name, focal_mm in ORDERS:
    half = math.atan(FLAME_RADIUS_M / (focal_mm / 1000))      # beam half-angle from fla
    omega = 2 * math.pi * (1 - math.cos(half))                # solid angle of the beam
    cd = CAPTURE * LUMENS / omega
    print(name, focal_mm, "mm", round(math.degrees(2 * half), 2), "deg", round(cd), "cd"
# Hyper-radial 1330 mm 0.86 deg 601451 cd 24.4 nm
# 1st 920 mm 1.25 deg 287801 cd 22.4 nm
# 2nd 725 mm 1.58 deg 178738 cd 21.2 nm
# 3rd 500 mm 2.29 deg 85025 cd 19.4 nm
# 4th 250 mm 4.58 deg 21275 cd 16.0 nm
# 6th 150 mm 7.63 deg 7675 cd 13.7 nm
```

The columns are the order, the focal length, the full width of the beam in degrees, the peak intensity and the nominal range at 10 nm visibility. The same burner in the hyper-radial lens makes a beam 0.86 degrees wide of about 601,000 cd and in a sixth-order lens a beam 7.63 degrees wide of about 7,700 cd. The light is the same; only the width of the beam changes. A large lens does not add light, it keeps it in a narrow beam. The model is a simplification: it treats the lens as a round lens with the flame at its focus and ignores the faults a real lens has. That is why a 10 cd flame comes out at hundreds of thousands of candela; a real light is not this tidy, and the figures show the trend, not any real lens.

Why a stepped lens is lighter

A solid lens of focal length f and glass index n needs surfaces of radius about (n − 1) × f. The thickness it adds in the middle (the sag) grows as the square of the aperture and gets worse as the focal length shortens. A stepped lens has the same optics in a flat panel of roughly constant thickness. The code compares the glass of a solid plano-convex lens with a flat panel 20 mm thick. The index, density, panel thickness and 5 mm edge are assumptions; the 920 mm focal length is the first-order figure from the table.

```
import math

N_GLASS = 1.5            # assumed refractive index
DENSITY = 2500.0         # kg per cubic metre, assumed for glass
```

```
FOCAL_M = 0.92           # first-order focal length from the Wikipedia table
STEP_THICK_M = 0.02      # assumed: a Fresnel panel about 2 cm thick

R = (N_GLASS - 1) * FOCAL_M        # radius of curvature of a plano-convex lens
for radius in (0.1, 0.2, 0.3, 0.4):
    sag = R - math.sqrt(R ** 2 - radius ** 2)                          # thickness added
    solid = math.pi * sag ** 2 * (3 * R - sag) / 3 + math.pi * radius ** 2 * 0.005   # c
    stepped = math.pi * radius ** 2 * STEP_THICK_M
    print(radius * 2, "m", round(sag * 1000, 1), "mm", round(solid * DENSITY, 1), "kg",
# 0.2 m 11.0 mm 0.8 kg 1.6 kg 0.5 x
# 0.4 m 45.8 mm 8.9 kg 6.3 kg 1.4 x
# 0.6 m 111.3 mm 44.7 kg 14.1 kg 3.2 x
# 0.8 m 232.8 mm 169.1 kg 25.1 kg 6.7 x
```

The columns are the diameter, the thickness of glass added in the middle, the mass of the solid lens, the mass of the stepped panel and the ratio of the two. At 0.2 m across, the stepped panel is *heavier* (1.6 kg against 0.8 kg for the solid one) because 20 mm is more than the solid lens needs at that size. Only above a diameter of about 0.33 m (the break-even found in Exercise 3) does stepping win: at 0.8 m the solid lens has a 233 mm bulge and weighs 169 kg against 25 kg, 6.7 times heavier, and the ratio keeps growing with size. The saving is largest exactly where a lighthouse needs it, in a lens a metre or more across. The thick middle of such a lens would also absorb light, the problem the article says Rogers's lenses had. These numbers come from the model: real panels have prisms and frames and weigh more than 20 mm of flat glass.

Common mistakes

Treating Condorcet's date as settled (1773, 1788 and 1790 all appear); saying Fresnel invented the stepped lens (Buffon described one in 1748); reading "85% against 20%" as "4.25 times the range" (the range in the model grew by about 1.3 times); assuming a larger lens makes more light (the same flame gives a narrower, brighter beam); assuming a stepped lens is always lighter (below about 0.33 m diameter in this model it is not); and quoting the model's intensities as those of a real lens (the burner, flame size and aperture are assumptions).

EXERCISE 1

Using 1,000 cd as the intensity of a 20% reflector, find the intensity of a 50% reflector and of an 85% lens, and the nominal range of each at visibilities of 5, 10 and 20 nm. Then find the ratio of the lens range to the reflector range at each visibility, and decide whether the "factor of four" quoted for Cordouan is a factor in intensity or in range.

View solution

EXERCISE 2

A 10 cd burner sits at the focus of a first-order lens (focal length 920 mm). Find the beam width, peak intensity and nominal range (10 nm visibility) for flame radii of 5, 10 and 20 mm. Then find the flame radius that gives a 2 degree beam, and the ratio of the intensities for a 10 mm flame in the hyper-radial lens (1330 mm) and the first-order lens.

View solution

EXERCISE 3

Compare the mass of a solid lens and of a 20 mm stepped panel at diameters of 0.2 m and 0.4 m for focal lengths of 0.5, 0.92 and 1.33 m. Then use a bisection search to find the diameter at which the two have the same mass for a focal length of 0.92 m, and explain why the break-even is not the same for every focal length.

View solution

QUICK REFERENCE

- **Reflector:** parabolic mirror; focused about 20% of a lamp's light (lighthouse article); Fresnel's reflectors reflected "about half" (Fresnel lens article)
- **Fresnel lens:** focused about 85% of the light; Cordouan lit 25 July 1823, visible more than 32 km (20 mi); luminosity up by a factor of four (lighthouse article)
- **Buffon, 1748:** a convex lens replaced by steps in one piece of glass; Condorcet's date is 1773, 1788 or 1790 by different sources
- **Dioptric / catadioptric:** refraction only / two refractions and one total internal reflection, for oblique light
- **Orders:** first order 920 mm focal length, hyper-radial 1330 mm, sixth 150 mm; a first-order assembly about 2.54 m tall and 1.5 tonnes
- **Model:** beam width from flame radius over focal length; a longer focal length gives a narrower, brighter beam, not more light; stepping saves glass only above a certain size

COMING NEXT

How Lighthouses Work 5 is about rotation and flashes: clockwork and weights, mercury float bearings, flash characters, light characteristics (Fl, Oc, Iso), and timing patterns in Python.

How Lighthouses Work

Chapter 5 · Rotation and Flashes

How Lighthouses Work 4 ended with a lens that squeezes the light of a small flame into a narrow beam. A narrow beam solves one problem and creates another: a ship sees it only when it points at the ship. The answer is to turn the lens, so that the beam sweeps the sea like the hand of a clock. Turning also solves a second problem for free. If every lighthouse showed a steady glow, a sailor could not tell one from the next. A rotating beam, and a lamp that is shuttered or switched, can give each light its own rhythm. This chapter covers the machinery that turns the lens and the shorthand that names the rhythm.

WHAT WAS RUN, AND WHAT TO TRUST

Every Python snippet in this chapter and in the three exercise solutions was run, and the output in the comments is what Python printed. The facts about clockwork, mercury, panel counts and flash rates were taken from the Wikipedia pages on lighthouses, on Fresnel lenses and on light characteristics, as fetched; where the pages disagree, the chapter says so. The *assumptions* are mine and are labelled where they appear: 5 W to turn a mercury-floated lens, four times that for a dry bearing, the example timings of each characteristic (for instance 1 s of light and 7 s of dark), and the beam widths carried over from How Lighthouses Work 4. The characteristics shown are illustrative patterns, not the timings of any named lighthouse. The 480 second turn in the first snippet is my arithmetic from the source's flash rates, not a figure the source states.

Turning the Lens

The lighthouse article says William Hutchinson developed the first practical optical system in 1777, which made the first revolving beams possible, and that Robert Stevenson's Bell Rock Lighthouse (1810) used rotating lights alternating between red and white. The Fresnel lens article describes Fresnel's next lens, a rotating apparatus with eight bull's-eye panels that gave eight rotating beams, and an 1825 plan for one flash per minute from a rotating lens with eight panels and two per minute from one with 16. A first-order catadioptric lens in Paris is described as giving four revolving beams, seen as four flashes per revolution; another lens gives three double flashes per rotation.

The sums are simple. With N panels equally spaced, the lens makes N flashes per turn, so the interval between flashes is the turn time divided by N. Run the plan backwards: eight panels and

one flash per minute need 8 minutes per turn, and 16 panels and two flashes per minute need the same 8 minutes. That is my reading of the source's numbers, not something it states. How long does an observer see each flash? The beam crosses the observer in the time it takes to turn through its own width.

```
# Flash interval, revolution period and how long one beam sweeps past an observer.
PLANS = (("8 panels, 1 flash per minute", 8, 1.0), ("16 panels, 2 flashes per minute", 1(

def revolution_s(panels, flashes_per_min):
    return panels * 60.0 / flashes_per_min

def dwell_s(beam_deg, period_s):
    return beam_deg / 360.0 * period_s

for name, panels, rate in PLANS:
    t = revolution_s(panels, rate)
    print(name, "->", t, "s per turn,", round(360 / t, 2), "deg per second")
for beam in (0.86, 1.25, 7.63):
    print(beam, "deg beam:", round(dwell_s(beam, 480), 2), "s at 480 s per turn,", round
print("beam width for a 1 s flash at 10 s per turn:", 1.0 / 10 * 360, "deg")
# 8 panels, 1 flash per minute -> 480.0 s per turn, 0.75 deg per second
# 16 panels, 2 flashes per minute -> 480.0 s per turn, 0.75 deg per second
# 0.86 deg beam: 1.15 s at 480 s per turn, 0.024 s at 10 s per turn
# 1.25 deg beam: 1.67 s at 480 s per turn, 0.035 s at 10 s per turn
# 7.63 deg beam: 10.17 s at 480 s per turn, 0.212 s at 10 s per turn
# beam width for a 1 s flash at 10 s per turn: 36.0 deg
```

The first two lines give the turn time (480 s) and the beam's angular speed (0.75 degrees per second). The next three give the time a ship sees a beam of 0.86, 1.25 and 7.63 degrees (the hyper-radial, first-order and sixth-order beams of the model in How Lighthouses Work 4) at 480 s and at 10 s per turn. The last line gives the beam width a 1 second flash would need at 10 s per turn: 36 degrees. This exposes a tension that I could not resolve from my sources. A fast-turning, narrow-beam lens gives a very brief flash (0.035 s for a 1.25 degree beam at 10 s per turn), yet the characteristics in the next section are described with flashes of a second or more. Either those lights have wide beams, or turn very slowly, or the figure is the effective duration of a flash as the eye sees it rather than the geometric one. I do not know which, so treat the flash length as the stated characteristic and the 0.035 s as the model's geometry.

Clockwork, weights and mercury

The lighthouse article says the lenses were rotated by a weight-driven clockwork assembly wound by keepers, sometimes as often as every two hours, and that the lens assembly sometimes floated in liquid mercury to reduce friction. It gives no date for the mercury. Later lighthouses used electric motors, generally powered by diesel generators, again with no date. How heavy would the weight have to be? It stores energy as mass times gravity times drop, and the lens uses power continuously, so the weight needed for a given winding interval depends on the drop. Here is the sum for the 2 hour winding, with an assumed 5 W for a floated lens and four times that for a dry bearing (both assumed):

```
# Weight-driven clockwork: how heavy a weight, how far it drops.
G = 9.81

def weight_kg(power_w, hours, drop_m):
    energy_j = power_w * hours * 3600
    return energy_j / (G * drop_m)

POWER_W = 5.0        # assumed: power to turn a mercury-floated lens
DRY_FACTOR = 4.0     # assumed: a dry bearing needs four times the power
WIND_H = 2.0         # the lighthouse article: sometimes as often as every two hours
for drop in (4, 8, 12):
    print(drop, "m drop:", round(weight_kg(POWER_W, WIND_H, drop)), "kg floated,", round
print("energy per 2 h at 5 W:", POWER_W * WIND_H * 3600, "J")
print("hours from a 300 kg weight falling 8 m:", round(300 * G * 8 / (POWER_W * 3600), 1
# 4 m drop: 917 kg floated, 3670 kg dry
# 8 m drop: 459 kg floated, 1835 kg dry
# 12 m drop: 306 kg floated, 1223 kg dry
# energy per 2 h at 5 W: 36000.0 J
# hours from a 300 kg weight falling 8 m: 1.3
```

The first three lines give the weight for 4, 8 and 12 m drops. A floated lens needs 459 kg at 8 m and a dry one 1,835 kg: four times the power means four times the weight for the same winding interval. The 36,000 J is the energy used in 2 hours at 5 W. The last line shows that a 300 kg weight falling 8 m would run only 1.3 hours at 5 W. The mercury therefore cuts the weight, or lengthens the winding interval, by a large factor in this model. The lens itself weighs tonnes (How Lighthouses Work 4 gives about 1.5 tonnes for a first-order assembly), so floating it is a sensible way to make it turn at all. Mercury is also toxic, which is general knowledge and not

something I read in the sources; it is a reason to prefer motors today, but that is my inference, not a quoted one.

DRIVE	WHAT THE SOURCE SAYS	WHAT IT NEEDS
Weight-driven clockwork	wound by keepers, sometimes every two hours	a keeper awake and winding through the night
Mercury float bearing	lens sometimes floated in liquid mercury to reduce friction	far less force to turn the lens (model: one quarter, assumed)
Electric motor	used in more modern lighthouses, often with a diesel generator	a power supply, not a keeper's arm

Flash Characters

A **light characteristic** is the pattern of light and dark that identifies a light. The Wikipedia article on light characteristics gives these abbreviations (colour letters W, G, R, Y and Bu follow, and white is implied if none is given):

ABBREVIATION	NAME	DEFINITION (FROM THE ARTICLE)
F	Fixed	a continuous and steady light
Fl	Flashing	a flash of light repeated, with a longer period of darkness
LFl	Long flashing	the flash lasts 2 to 5 seconds
Q, VQ, UQ	Quick, very quick, ultra quick	a flashing light of high frequency (see the conflict below)
Iso	Isophase	dark and light periods of equal length
Oc	Occulting	the light in each period lasts longer than the total darkness
Mo	Morse code	two clearly different durations of light, dots and dashes
Al	Alternating	a light showing alternating colours

Source conflict on the quick lights: the article says the threshold for "quick" is given as either 30 or 50 flashes per minute depending on the source, and elsewhere it gives bands of 50 to 80 flashes per minute for quick, 80 to 159 (in another place "more than 80 and less than 160") for very quick, and 160 or more for ultra quick. I have not resolved which threshold is used where.

The article also gives rules for a single flash: a period of 2 to 15 seconds (2.5 to 15 in countries that use quick lights) and a darkness of at least three times the flash. A group of flashes is written with the count in brackets, so Fl(2) is two flashes per group, with a period of no more than 20 seconds for two flashes and no more than 30 for three or more. The article's example, Fl(3) R 15s, is three red flashes of 1.0 s separated by 1.0 s of darkness, then 10.0 s of darkness, a 15 second cycle. What do these look like? The code draws each one as a row of characters, a hash for light and a dot for dark, in half-second steps:

```
# Light characteristics as timelines, drawn at half-second steps.
CHARS = {
    "Fl W 5s":     [("L", 1), ("D", 4)],
    "LFl W 12s":   [("L", 3), ("D", 9)],
    "Fl(2) W 10s": [("L", 1), ("D", 1), ("L", 1), ("D", 7)],
    "Fl(3) R 15s": [("L", 1), ("D", 1), ("L", 1), ("D", 1), ("L", 1), ("D", 10)],
    "Iso W 4s":    [("L", 2), ("D", 2)],
    "Oc W 5s":     [("L", 4), ("D", 1)],
    "Mo(A) W 12s": [("L", 1), ("D", 1), ("L", 3), ("D", 7)],
}

def period(segs):
    return sum(d for _, d in segs)

def light_share(segs):
    return sum(d for s, d in segs if s == "L") / period(segs)

def draw(segs, window_s=30, step=0.5):
    out = ""
    for i in range(int(window_s / step)):
        t = (i * step) % period(segs)
        acc = 0.0
        for s, d in segs:
            acc += d
            if t < acc:
                out += "#" if s == "L" else "."
                break
    return out

def check_single_flash(segs):
    lights = [d for s, d in segs if s == "L"]
    darks = [d for s, d in segs if s == "D"]
    return period(segs) >= 2 and period(segs) <= 15 and darks[0] >= 3 * lights[0]
```

```
for name, segs in CHARS.items():
    print(name.ljust(12), draw(segs), round(light_share(segs), 2))
print("Fl W 5s passes the single-flash rules:", check_single_flash(CHARS["Fl W 5s"]))
print("Fl(3) R 15s period:", period(CHARS["Fl(3) R 15s"]), "s")
# Fl W 5s      ##........##........##........##........##........##........ 0.2
# LFl W 12s    ######..................######..................######...... 0.25
# Fl(2) W 10s  ##..##..............##..##..............##..##.............. 0.2
# Fl(3) R 15s  ##..##..##....................##..##..##.................... 0.2
# Iso W 4s     ####....####....####....####....####....####....####....#### 0.5
# Oc W 5s      ########..########..########..########..########..########.. 0.8
# Mo(A) W 12s  ##..######..............##..######..............##..######.. 0.33
# Fl W 5s passes the single-flash rules: True
# Fl(3) R 15s period: 15 s
```

Each row shows the characteristic, 30 seconds of its pattern, and the share of the time the light is on. The Fl(3) R 15s row follows the article's example exactly (three flashes, gaps of 1 s and a 10 s pause, so 3 + 2 + 10 = 15). The other timings are my illustrative choices: Fl 5s with a 1 s flash passes the "at least three times" darkness rule (4 s against 1 s), LFl has a 3 s flash (within the 2 to 5 s range), and Mo(A) is a 1 s dot and a 3 s dash (a short and a long signal, the letter A; the 12 s period is my choice). The isophase light is on half the time and the occulting one 80 per cent of the time, while the flashing lights are on for a fifth to a third. A sailor can look away, time the pattern with a watch and compare it with the chart, as How Lighthouses Work 9 does when it reads one.

From Pattern to Panel

How does a lens that only turns make Fl(2) or Fl(3)? The panels are placed in pairs or threes, and the angle at which each flash starts is its start time divided by the period, times 360 degrees. The code works out where each flash starts and how wide it must be, from the same illustrative timelines:

```
# Where the flash panels must sit on a rotating lens.
CHARS = {
    "Fl(2) W 10s": [("L", 1), ("D", 1), ("L", 1), ("D", 7)],
    "Fl(3) R 15s": [("L", 1), ("D", 1), ("L", 1), ("D", 1), ("L", 1), ("D", 10)],
}
```

```
def panel_layout(segs):
    total = sum(d for _, d in segs)
    t = 0.0
    out = []
    for s, d in segs:
        if s == "L":
            out.append((round(t / total * 360, 1), round(d / total * 360, 1)))
        t += d
    return out

for name, segs in CHARS.items():
    print(name, "(start deg, width deg):", panel_layout(segs))
# Fl(2) W 10s (start deg, width deg): [(0.0, 36.0), (72.0, 36.0)]
# Fl(3) R 15s (start deg, width deg): [(0.0, 24.0), (48.0, 24.0), (96.0, 24.0)]
```

For Fl(2) W 10s the two flashes start at 0 and 72 degrees and each is 36 degrees wide (a 1 s flash on a 10 s turn). For Fl(3) R 15s the three start at 0, 48 and 96 degrees and are 24 degrees wide. The widths are large because the flashes last a whole second, which is the tension described in the first section: the model implies beams of tens of degrees for second-long flashes. I could not check this against a real lens, and the Fresnel article's own examples (three double flashes per rotation) describe how many flashes there are, not their angular size. The method (angles from times) is plain arithmetic; the numbers are the model's.

> Common mistakes
> Reading "one flash per minute from eight panels" as one turn per minute (it is eight minutes per turn, by my arithmetic); treating the mercury as a source of light rather than a bearing; counting an occulting light (long light, brief dark) as a flashing one; applying the "darkness three times the flash" rule to every characteristic (the article states it for single flashes, and I applied it only there); forgetting that the quick-light threshold is quoted as both 30 and 50 flashes per minute; and quoting the example timings in this chapter as those of a real light.

EXERCISE 1

Write a function that classifies a light from timed observations of one repeat of its pattern (a list of light and dark durations in seconds) as Fl, LFl, Oc, Iso, Mo or Fl(n), and returns the period. Run it on six observed patterns, print the share of the time the light is on, and check the plain flash against the rule that darkness must be at least three times the flash.

View solution

EXERCISE 2

A lens has 4 equally spaced panels and turns once in 20 s. Find the flash interval and the rate per minute, the time a 1.25 degree beam is seen, the turn time that would give a 5 s interval with 4 panels, the angle between the two flashes of an Fl(2) 10s pair, and the beam width needed for a 1 s flash at 20 s and 5 s per turn. Finally find how fast the beam sweeps at a ship 10 nm away.

View solution

EXERCISE 3

With 5 W to turn a floated lens and four times that dry, find the weight needed for an 8 m drop at winding intervals of 2 and 4 hours, then how long a 300 kg weight with a 10 m drop runs floated and dry, and how many windings a 12 hour night needs at each interval.

View solution

QUICK REFERENCE

- **Rotation:** N equally spaced panels give N flashes per turn; flash interval is turn time divided by N; the source's 1825 plan (8 panels, one flash per minute) works out to 8 minutes per turn
- **Drive:** weight-driven clockwork wound by keepers, sometimes every two hours; lens sometimes floated in liquid mercury to reduce friction; later electric motors
- **Characteristics:** F fixed, Fl flashing, LFl 2 to 5 s flash, Iso equal light and dark, Oc light longer than dark, Mo Morse, Al alternating colours
- **Rules:** single flash period 2 to 15 s with darkness at least three times the flash; groups written Fl(n); example Fl(3) R 15s = 3 flashes of 1 s, 1 s gaps, 10 s dark
- **Conflict:** the quick threshold is 30 or 50 flashes per minute depending on the source
- **Model:** flash start angle = start time divided by period times 360; beam width for a flash = flash time divided by period times 360

COMING NEXT

How Lighthouses Work 6 is about building on rock: Bell Rock (1810), Skerryvore (1844), Muckle Flugga, tide-limited working, interlocking stones, and wave forces.

How Lighthouses Work

Chapter 6 · Building on Rock

So far the course has dealt with the light: how far it carries, what fuel feeds it, how a lens shapes it and how a turning lens gives it a rhythm. None of it matters if the tower is not there. The most useful places for a lighthouse are often the worst places to build one: a reef that is covered at high tide, a rock far out in the Atlantic, a stack at the northern tip of Britain. How Lighthouses Work 2 followed Smeaton's tower on the Eddystone, the first to be thought through as engineering. This chapter follows three later towers on Scottish rocks, Bell Rock (first lit in 1811), Skerryvore (1844) and Muckle Flugga (1858), and asks three practical questions of each. When can anyone work there? How is a stone tower held together? And what do the waves do to it?

WHAT WAS RUN, AND WHAT TO TRUST

Every Python snippet in this chapter and in the three exercise solutions was run, and the output in the comments is what Python printed. The facts about the three lighthouses were taken from the Wikipedia pages on Bell Rock Lighthouse, Skerryvore, Muckle Flugga and Muckle Flugga Lighthouse, fetched through a page-reading tool that answers questions about a page, so the figures are as that tool reported them and I could not check them against the printed sources the pages cite. Where a page disagrees with itself, the chapter says so. The *assumptions* are mine and are labelled where they appear: a tide cycle of 12.42 hours, half of the exposed time being usable, a seawater density of 1,025 kg per cubic metre, 16 hour working days for the stone cutters, and the eye heights used for the range checks. The conversions between tons, tonnes, feet and metres, and the ratios built on them, are my arithmetic. Nothing here is a record of how any tower was actually calculated.

Bell Rock: Working Between the Tides

The Bell Rock page gives the construction as 1807 to 1810 under Robert Stevenson, with the light first lit on 1 February 1811. The rock lies about 11 miles (18 km) east of the Firth of Tay and is covered by water for about 20 hours a day, with up to 12 feet (3.7 m) of water over it at high tide. That one number sets the whole project: only about four hours of every day are available, and they move. Stevenson hired 60 men in 1807, including a blacksmith to resharpen pickaxes on the rock, and avoided black powder so as not to damage it. Early on the men lived on a ship moored about a mile away and rowed out each day; then a beacon house on wooden struts was built, with room for up to 15 men. Granite from Cairngall was cut by stonemasons in the winter pauses. The first three courses were laid in early 1808, and the article says only 80 hours of

building work took place on the rock during that second season. It puts the whole job at about 2,500 granite stones, hauled by one horse named Bassey.

How much work is 80 hours? The code turns the 20 hours of cover into windows of working time and asks what the 2,500 stones would need at some assumed laying rates. The tide period, the usable share and the stones per hour are my assumptions; the source gives the cover, the 80 hours and the stone count.

```
# Tide-limited working on a rock that is covered for most of the day.
COVERED_H_PER_DAY = 20.0   # the Bell Rock article: covered by water for about 20 hours
TIDE_PERIOD_H = 12.42      # assumed: a typical twice-daily tide cycle (general knowledg
SECOND_SEASON_H = 80       # the article: hours of building work on the rock in the seco
USABLE_SHARE = 0.5         # assumed: only half of the exposed time is usable (ladders,
TOTAL_STONES = 2500        # the article: "about 2500" granite stones

exposed_h = 24 - COVERED_H_PER_DAY
windows_per_day = 24 / TIDE_PERIOD_H
print("exposed per day:", exposed_h, "h, in", round(windows_per_day, 2), "windows of", r
print("daily slip of the tide:", round((2 * TIDE_PERIOD_H - 24) * 60), "min")
print("days to collect 80 h at all exposed time:", SECOND_SEASON_H / exposed_h)
print("days to collect 80 h at 50 per cent usable:", SECOND_SEASON_H / (exposed_h * USAB
for rate in (2, 4, 8):
    hours = TOTAL_STONES / rate
    print(rate, "stones per working hour:", hours, "h in all =", round(hours / SECOND_SE
# exposed per day: 4.0 h, in 1.93 windows of 2.07 h
# daily slip of the tide: 50 min
# days to collect 80 h at all exposed time: 20.0
# days to collect 80 h at 50 per cent usable: 40.0
# 2 stones per working hour: 1250.0 h in all = 15.6 seasons of 80 h
# 4 stones per working hour: 625.0 h in all = 7.8 seasons of 80 h
# 8 stones per working hour: 312.5 h in all = 3.9 seasons of 80 h
```

The first two lines show four exposed hours a day in 1.93 windows of about 2.07 hours, and a tide that arrives about 50 minutes later each day (twice 12.42 hours, less 24). The next two give the days needed to collect 80 hours: 20 if every exposed minute could be used, 40 if only half could. The last three lines say that 2,500 stones at 2, 4 or 8 stones per working hour need 1,250, 625 or 312.5 hours, which is 15.6, 7.8 or 3.9 seasons of 80 hours. The article says the tower was finished in 1810, three years after starting, so the later seasons must have had far more working

time than the second one, or the laying rate was high, or the stone count is not complete. I do not know which; the article gives no total hours and no laying rate. Two statements that use only the article's own numbers are safe: the rock gave about four hours a day, and the second season gave 80 hours.

PROBLEM AT THE BELL ROCK	WHAT THE ARTICLE SAYS	WHAT IT MEANT
Cover	under water about 20 hours a day, up to 12 ft at high tide	a few hours a day of work, at times set by the tide
Housing	a ship moored 1 mile away, then a beacon house on struts for up to 15 men	men on or near the rock, so no long trip each tide
Rock	no black powder, to avoid damaging it	picks and chisels, and a blacksmith on the spot
Stone	Cairngall granite, cut in the winter pauses	shaping on land, only setting on the rock

Interlocking Stones

The Bell Rock article says the design was based on John Smeaton's Eddystone Lighthouse, which Stevenson studied in 1801, and that Eddystone was also built on an offshore reef using interlocking stones. It quotes a historian describing the tower as "a copy of Smeaton's Eddystone Tower with sundry improvements in details and dimensions". It does not say how the Bell Rock blocks were joined. How Lighthouses Work 2 gave the Smeaton tower article's account: granite blocks joined with dovetail joints and marble dowels, with hydraulic lime. The Bell Rock page does not repeat any of this, so I cannot say which of Smeaton's methods Stevenson kept. There is also a conflict inside the Bell Rock page about who held the contract: the text says it was awarded to John Rennie with Stevenson as chief assistant, while the infobox credits Stevenson as builder.

Skerryvore's page is more generous with numbers, though it does not give the joint either. It says the tower was designed and built under Alan Stevenson; work on the rock began on 7 August 1838; the first block was laid on 4 July 1840 by the Duke of Argyll; the last block was raised in July 1842; and the lamp was lit on 1 February (the year is implied to be 1844). The first three courses are Hynish gneiss and the rest granite from the Ross of Mull. Up to 80 craftsmen worked on the blocks at Hynish on Tiree over the winter of 1840, from 4 am to 8 pm. The tower

weighs 4,308 long tons (given as 4,377 tonnes), 4,300 blocks were fashioned by June 1840, one block could take 320 worker-hours, and the smallest block weighed 0.75 long tons and the largest over 2.5. The code checks the units and asks what these numbers can and cannot mean.

```
# Skerryvore masonry: units, block sizes and the cost of cutting stone.
LT_TO_T = 1.0160469        # one long ton in tonnes
STRUCTURE_LT = 4308        # the Skerryvore article: weight of the structure
BLOCKS_BY_JUN_1840 = 4300  # blocks fashioned by June 1840
SMALLEST_LT, LARGEST_LT = 0.75, 2.5
ROCK_REMOVED_LT = 2000
MAX_WORKER_H = 320         # worker-hours that one block could take
CRAFTSMEN = 80             # up to 80 craftsmen at Hynish in the winter of 1840
DAY_H = 16                 # 4 am to 8 pm

print("structure:", round(STRUCTURE_LT * LT_TO_T), "t")
print("rock removed:", round(ROCK_REMOVED_LT * LT_TO_T), "t (the article prints 2,000 to
print("block range:", round(SMALLEST_LT * LT_TO_T, 2), "to over", round(LARGEST_LT * LT_
avg_t = STRUCTURE_LT * LT_TO_T / BLOCKS_BY_JUN_1840
print("mean mass if all the blocks made up the tower:", round(avg_t, 2), "t")
wh_per_day = CRAFTSMEN * DAY_H
print("worker-hours per day:", wh_per_day, ", blocks per day at 320 h each:", wh_per_day
print("days for 4300 blocks at 320 h each:", round(BLOCKS_BY_JUN_1840 * MAX_WORKER_H / wh
for days in (100, 200, 300):   # assumed lengths of the cutting season
    print(days, "days of cutting: mean", round(wh_per_day * days / BLOCKS_BY_JUN_1840, 1
# structure: 4377 t
# rock removed: 2032 t (the article prints 2,000 tonnes)
# block range: 0.76 to over 2.54 t
# mean mass if all the blocks made up the tower: 1.02 t
# worker-hours per day: 1280 , blocks per day at 320 h each: 4.0
# days for 4300 blocks at 320 h each: 1075
# 100 days of cutting: mean 29.8 worker-hours per block
# 200 days of cutting: mean 59.5 worker-hours per block
# 300 days of cutting: mean 89.3 worker-hours per block
```

The first line confirms the conversion: 4,308 long tons is 4,377 tonnes, so the page is consistent there. The second line shows that the rock removal figure is slightly off: 2,000 long tons is 2,032 tonnes, not the 2,000 tonnes the page prints (a long ton is 1.016 tonnes). Blocks ran from 0.76 to over 2.54 tonnes. If the 4,300 blocks made up the whole tower, the mean block would weigh 1.02 tonnes, which is inside that range, but this is a plausibility check only: the page does not say

the 4,300 blocks were all of them. At 320 worker-hours per block, 80 craftsmen working 16 hour days (1,280 worker-hours a day) would finish 4 blocks a day and take 1,075 days for 4,300 blocks, far more than a winter. Over assumed cutting seasons of 100, 200 and 300 days, the mean cost per block must have been only 29.8, 59.5 or 89.3 worker-hours. So 320 hours is a maximum for an awkward block, not an average. The two similar numbers, 4,308 long tons and 4,300 blocks, are a coincidence of digits.

TOWER	HOW THE STONES ARE DESCRIBED	WHAT IS NOT STATED
Smeaton's Eddystone (1759)	granite blocks with dovetail joints and marble dowels, hydraulic lime, shaped like an oak tree (from How Lighthouses Work 2)	the strength of the joints
Bell Rock	based on Eddystone, which used interlocking stones; Cairngall granite	the joint type, wall thickness and base diameter
Skerryvore	tapered cylinder on a hyperbolic curve; Hynish gneiss in the lowest three courses, Ross of Mull granite above; blocks cut at Hynish	how the blocks are joined

What the Waves Do

The Skerryvore article gives three figures about the sea: a design wave pressure of 6,000 lb per square foot (printed as 300 kPa), waves reportedly reaching the first window, about 60 ft up, according to a visitor in 1866, and 542 storms lasting 14,211 hours in the years 1881 to 1890. It also says the tower is a tapered cylinder with walls 9.5 ft (2.9 m) thick at the base and a solid lower section 26 ft (8 m) high. What does a pressure of 6,000 lb per square foot mean? The code converts it, compares it to a block, and then tests a deliberately pessimistic idea: that the full pressure acts at once over the whole face up to 60 ft. The seawater density and the whole-face idea are my assumptions; the page does not say over what area the design pressure was meant to act.

```
# Wave pressure on Skerryvore: a pessimistic bound.
import math

LBF_N = 4.4482216       # newtons in one pound-force
FT_M = 0.3048
```

```
RHO_SEA = 1025.0        # assumed: density of seawater, kg/m3
G = 9.81
P_LB_FT2 = 6000         # the article: design wave pressure, lb per square foot
BASE_D_M = 13.0         # the article: base diameter 42 ft = 13 m
WAVE_H_M = 60 * FT_M    # the article: waves reportedly reached the first window, about
WEIGHT_T = 4377         # the article: structure, in tonnes

p = P_LB_FT2 * LBF_N / FT_M ** 2
print("pressure:", round(p / 1000, 1), "kPa (the article prints 300 kPa)")
print("equivalent head of seawater:", round(p / (RHO_SEA * G), 1), "m")
print("force on 1 square metre:", round(p / 1000), "kN =", round(p / 1000 / G, 1), "tonn
print("largest block:", round(2.5 * 1.0160469, 2), "t, ratio:", round(p / G / 1000 / (2.
area = BASE_D_M * WAVE_H_M
force = p * area
weight = WEIGHT_T * 1000 * G
print("projected area:", round(area), "m2, force if the whole face took it:", round(forc
print("weight:", round(weight / 1e6, 1), "MN, ratio force/weight:", round(force / weight
print("overturning moment:", round(force * WAVE_H_M / 2 / 1e6), "MN m, resisting:", roun
# pressure: 287.3 kPa (the article prints 300 kPa)
# equivalent head of seawater: 28.6 m
# force on 1 square metre: 287 kN = 29.3 tonnes-force
# largest block: 2.54 t, ratio: 11.5
# projected area: 238 m2, force if the whole face took it: 68.3 MN
# weight: 42.9 MN, ratio force/weight: 1.59
# overturning moment: 625 MN m, resisting: 279 MN m
```

The pressure is 287.3 kPa, so the printed 300 kPa is rounded. It is the pressure at about 28.6 m depth of seawater, and it puts 287 kN (29.3 tonnes-force) on a square metre, 11.5 times the weight of the largest block. So a stone on the face can never stand on its own weight; it has to be held by its neighbours, which is the point of interlocking blocks. Then the pessimistic test: over the whole 238 square metre face up to 60 ft the force would be 68.3 MN, which is 1.59 times the whole tower's weight of 42.9 MN, and the overturning moment (625 MN m) would be more than twice the resisting moment (279 MN m, taking the full weight acting at half the base diameter). A tower designed for that pressure would fall over if it acted everywhere at once, so it cannot be meant to. My reading is that the figure is a peak local pressure from a breaking wave, acting briefly on a small patch, though I found nothing in the source that says so. The tower is also bonded to the rock: the article says 2,000 long tons of rock were removed for the foundations, and I take that to mean the base was cut into the rock, which is an inference.

Muckle Flugga: the Same Sea, Higher Up

Muckle Flugga, a stack off Unst in Shetland, shows the sea at its worst from a different angle. The Muckle Flugga Lighthouse page says the brothers Thomas and David Stevenson designed and built it, that a temporary 50 ft (15 m) lighthouse was built 200 ft (61 m) above sea level and lit on 11 October 1854, that orders to build the permanent tower were given in June 1855, and that it was completed in 1857 and first lit on 1 January 1858. It records that "water was known to pass at a height of 200 feet" and that Stevenson saw "a huge six tonne piece of rock that had been removed by the waves, eighty feet above the sea". Winter storms sent waves onto the temporary tower and broke open the door of its living quarters. Bricks were chosen because they were easier to transport. Supplies were winched up by a Blondin cable hoist from a boat in a cleft of the rock, and every item for the permanent tower had to be carried to the top. The page also conflicts with itself on the height ("64 feet (20 m)" in the text, "20 m (66 ft)" in the infobox) and on the material (the infobox says masonry, the history says bricks). The island page says the light was automated in 1995.

Three Rocks Compared

	BELL ROCK	SKERRYVORE	MUCKLE FLUGGA
Site	reef about 11 miles east of the Firth of Tay	about 11 nautical miles southwest of Tiree	stack off Unst, Shetland
Built	1807 to 1810, first lit 1 Feb 1811	rock work from 1838, lit 1 Feb (1844)	temporary light 1854, tower 1855 to 1857, lit 1 Jan 1858
Height	35 m in the text, 36 m in the infobox	48 m (156 ft in the text, 157 ft in the infobox)	20 m ("64 feet" in the text, "66 ft" in the infobox)
Focal height	28 m	46 m	66 m
Characteristic	Fl W 5s	Fl W 10s	Fl(2) W 20s
Listed range	18 nm (the lead also says 35 statute miles)	23 nm	22 nm
Main difficulty	tide: covered about 20 hours a day	exposure: storms far from shore, and a barrack lost in a gale	height and access: waves reported at 200 ft, hoists
Cost	estimate £42,000, about 50 per cent over	£86,977 including about £13,000 for Hynish	not given

	BELL ROCK	SKERRYVORE	MUCKLE FLUGGA
Automated	1988	1994	1995

The characteristics are the kind described in How Lighthouses Work 5: the periods are 5, 10 and 20 seconds, and Muckle Flugga is a group flash. Skerryvore's distance from Tiree is given as 11 nautical miles (20 km) in the lead, 12 miles from Hynish elsewhere and 14 miles in an 1814 account, and it is also described as roughly 50 miles from the mainland; I have used the lead's figure. The page also says a wooden barrack on the rock for 40 men was destroyed by a gale on 3 November 1838 and replaced by early September 1839, and that Hynish on Tiree was the shore station and construction base. The Bell Rock cost of about £63,000 is not stated by the article; it is my arithmetic from a 50 per cent overrun on £42,000. Do the listed ranges agree with the geometry of How Lighthouses Work 1? The code asks, for each light, what eye height would give exactly the listed range, using 2.084 nautical miles per square root of a metre.

```
# Do the quoted ranges fit the geometry? Eye height each listed range would need.
import math

NM_PER_SQRT_M = 2.084   # from How Lighthouses Work 1: 3.86 km divided by 1.852 km per nm
LIGHTS = (              # (name, focal height m, listed range nm), from each lighthouse's
    ("Bell Rock", 28, 18),
    ("Skerryvore", 46, 23),
    ("Muckle Flugga", 66, 22),
)

def geographic_range_nm(light_m, eye_m):
    return NM_PER_SQRT_M * (math.sqrt(light_m) + math.sqrt(eye_m))

def eye_needed_m(light_m, range_nm):
    return (range_nm / NM_PER_SQRT_M - math.sqrt(light_m)) ** 2

for name, height, listed in LIGHTS:
    print(name, "| range for a 5 m eye:", round(geographic_range_nm(height, 5), 1), "nm
print("66 m in feet:", round(66 / 0.3048), "; 64 ft in m:", round(64 * 0.3048, 1), "; 66
print("six tonnes lifted 80 ft:", round(6000 * 9.81 * 80 * 0.3048 / 1e6, 2), "MJ")
# Bell Rock | range for a 5 m eye: 15.7 nm | listed: 18 nm | eye needed: 11.2 m
# Skerryvore | range for a 5 m eye: 18.8 nm | listed: 23 nm | eye needed: 18.1 m
# Muckle Flugga | range for a 5 m eye: 21.6 nm | listed: 22 nm | eye needed: 5.9 m
# 66 m in feet: 217 ; 64 ft in m: 19.5 ; 66 ft in m: 20.1
```

```
# six tonnes lifted 80 ft: 1.44 MJ
```

For an eye 5 m up, the geographic ranges are 15.7, 18.8 and 21.6 nm, against listed ranges of 18, 23 and 22. Muckle Flugga is consistent: it needs an eye only 5.9 m up. Bell Rock needs 11.2 m and Skerryvore 18.1 m, so either those ranges assume a high bridge or deck, or they are not geographic ranges at all (the Bell Rock article gives both 18 nautical miles and 35 statute miles inland, and I did not find what each means). I could not resolve this from my sources, so treat the listed ranges as the pages' numbers, not checked ones. The next-to-last line shows 66 m is 217 ft (matching the infobox focal height in both units), and that 64 ft is 19.5 m while 66 ft is 20.1 m, so the two Muckle Flugga heights differ by 0.6 m. The last line gives 1.44 MJ as the work needed just to lift the six tonne rock 80 ft, which is a lower bound on what the wave did to it.

Common mistakes

Treating 4,308 long tons and 4,300 blocks as the same quantity; mixing long tons, short tons and tonnes (they differ by up to 12 per cent); treating the 320 worker-hours for one block as an average; reading the Skerryvore design pressure as acting on the whole tower at once; assuming the Bell Rock joined its stones in the same way as the Eddystone (the page does not say); trusting the listed ranges of the three lights without asking how they were defined; taking the Bell Rock overrun of about £63,000 as a figure from the page; and taking the page's "2,000 tonnes" of foundation rock at face value, when the source figure is in long tons.

EXERCISE 1

25, 50 and 75 per cent of the time is usable, and the number of such seasons that 2,500 stones need at 3 stones an hour. Then print the time a low-water window opens on each of seven days, starting at 06:00, if the tide slips by two tide periods of 12.42 hours less 24 hours a day, and the number of days before the clock time repeats.

View solution

EXERCISE 2

Using Skerryvore figures, convert the smallest (0.75 long tons) and largest (2.5 long tons) blocks to tonnes, find how many blocks of each size the 4,308 long ton tower would be, convert the base wall (9.5 ft), top wall (2 ft) and solid section (26 ft) to metres, and give the wall-to-diameter and solid-to-

height shares. Then find how many blocks 80 craftsmen make in a 6-day week of 16 hour days at 30, 60 and 120 worker-hours per block.

View solution

EXERCISE 3

Convert wave pressures of 1,000, 3,000 and 6,000 lb per square foot to kilopascals, to the head of seawater, and to tonnes-force on one square metre. Then find the height of light needed for a 22 nm geographic range with an eye 5 m and 10 m above the sea, and convert the two Muckle Flugga heights, 64 ft and 66 ft, to metres.

View solution

QUICK REFERENCE

- **Bell Rock:** built 1807 to 1810, first lit 1 Feb 1811; covered about 20 hours a day; about 2,500 granite stones; based on Smeaton's Eddystone; Fl W 5s, focal height 28 m
- **Skerryvore:** Alan Stevenson; rock work from 7 Aug 1838; 4,308 long tons (4,377 tonnes); base diameter 42 ft (13 m); 4,300 blocks by June 1840; Fl W 10s, focal height 46 m
- **Muckle Flugga:** Thomas and David Stevenson; temporary light 1854, tower first lit 1 Jan 1858; Fl(2) W 20s, focal height 66 m; supplies by Blondin hoist
- **Waves:** design pressure 6,000 lb per square foot = 287.3 kPa (printed as 300 kPa) = 29.3 tonnes-force per square metre; probably a local peak, not a uniform load (my reading)
- **Units:** 1 long ton = 1.0160469 tonnes; 1 ft = 0.3048 m; 2.084 nm per square root of a metre
- **Conflicts:** Bell Rock height 35 or 36 m and contractor Rennie or Stevenson; Skerryvore height 156 or 157 ft; Muckle Flugga height 64 or 66 ft and masonry or brick; listed ranges versus geometry

COMING NEXT

How Lighthouses Work 7 is about the Stevensons: Robert Stevenson, Alan, David and Thomas, the Northern Lighthouse Board, Robert Louis Stevenson's family link, and the wave dynamometer.

How Lighthouses Work

Chapter 7 · The Stevensons

Three towers in How Lighthouses Work 6 had one name running through them: Stevenson. Robert built the Bell Rock, his son Alan built Skerryvore, and his sons David and Thomas built Muckle Flugga. This chapter steps back from the stone to the people. Who were they, how did one family come to supply the engineers of a whole country's lights for most of a century, how did one of them try to measure what the sea actually does to a tower, and what did the writer in the family make of it all? It is a story about engineering, but also about a body, the Northern Lighthouse Board, that kept the work going from one generation to the next.

WHAT WAS RUN, AND WHAT TO TRUST

Every Python snippet in this chapter and in the three exercise solutions was run, and the output in the comments is what Python printed. The facts were taken from the Wikipedia pages on Robert Stevenson (civil engineer), Alan Stevenson, David Stevenson (engineer), Thomas Stevenson, Robert Louis Stevenson, the Northern Lighthouse Board and Skerryvore, from a Trinity House article on wave loading of rock lighthouses, and from Alan Stevenson's 1848 *Account of the Skerryvore Lighthouse* as hosted by Project Gutenberg. All were read through a page-reading tool that answers questions about a page. That tool read only the first 100,000 characters of the very long Robert Louis Stevenson page and of the Gutenberg text, so I did not read either in full, and I could not check any figure against the printed sources those pages cite. Where a page disagrees with itself or with another page, the chapter says so. The *assumptions* are mine and are labelled: a seawater density of 1,025 kg per cubic metre, and a 3 inch disc in Exercise 2 (illustrative only). All sums, conversions and ratios are my arithmetic. Nothing here is a record of how any engineer actually calculated.

Four Engineers, One Board

The Northern Lighthouse Board was set up by the Erection of Lighthouses Act 1786, which the Board's page dates to royal assent on 27 June 1786 (its infobox says 1867, which conflicts with the Act, and I have taken the Act). It is the general lighthouse authority for Scotland and the Isle of Man; its headquarters is a Georgian building in George Street, Edinburgh, with technical operations in Oban, and as of 31 March 2024 it was responsible for 208 statutory lighthouses (65 with a range over 15 nautical miles and 143 under, which does add up). Its first four lights were Kinnaird Head, North Ronaldsay, Scalpay and the Mull of Kintyre. The same page calls Robert Stevenson the commissioners' most famous engineer and says his sons David, Alan and

Thomas followed him and that the family built most of the northern lights. It does not give the end of the family's tenure.

The family's link with the Board starts one step before Robert. His father, a West Indies sugar merchant, died in 1774, and his mother's second husband was Thomas Smith, a tinsmith, mechanic and civil engineer who joined the Board in 1786. Robert trained under Smith, married Smith's daughter Jean (so his stepsister) in 1799, and was appointed engineer to the Board in 1797, succeeding Smith. The code below records the dates the pages give, works out what can be worked out, and then follows the family tree from three younger members back to Smith.

```
# Four Stevensons: lifespans, board service and lighthouse counts, from the Wikipedia pa
PEOPLE = (   # (name, born, died, lighthouses on the person's page, first year, last yea
    ("Robert", 1772, 1850, None, None, None),  # the page lists "about 22"; I was not sh
    ("Alan",   1807, 1865, 13, 1843, 1853),    # 13 built 1843-1853 (the list has 14 ent
    ("David",  1815, 1886, 28, 1854, 1880),    # lead says "over 30"; the list has 28
    ("Thomas", 1818, 1887, 31, 1854, 1886),    # 31 listed, Whalsay Skerries to Ailsa Cr
)
for name, born, died, count, first, last in PEOPLE:
    line = name + " | lived " + str(died - born) + " years"
    if count:
        span = last - first + 1
        line += " | lights " + str(first) + " to " + str(last) + " | per year: " + str(r
    print(line)

ROBERT_NLB = (1797, 1842)
print("Robert's board service:", ROBERT_NLB[1] - ROBERT_NLB[0], "years (the page says 'n
print("years between Robert's service and Alan's start:", 1843 - ROBERT_NLB[1] - 1)
print("Robert's appointment to Thomas's last lighthouse:", 1886 - ROBERT_NLB[0], "years"

# Who is related to whom? Parent links taken from the pages.
PARENT = {"Robert": "Thomas Smith",      # stepfather, and also Robert's father-in-law
          "Alan": "Robert", "David": "Robert", "Thomas": "Robert",
          "R. L. Stevenson": "Thomas", "David Alan": "David", "Katharine": "Alan"}
def line_of(person):
    chain = [person]
    while chain[-1] in PARENT:
        chain.append(PARENT[chain[-1]])
    return chain
for p in ("R. L. Stevenson", "David Alan", "Katharine"):
    print(" -> ".join(line_of(p)))
# Robert | lived 78 years
# Alan | lived 58 years | lights 1843 to 1853 | per year: 1.18
```

```
# David | lived 71 years | lights 1854 to 1880 | per year: 1.04
# Thomas | lived 69 years | lights 1854 to 1886 | per year: 0.94
# Robert's board service: 45 years (the page says 'nearly fifty')
# years between Robert's service and Alan's start: 0
# Robert's appointment to Thomas's last lighthouse: 89 years
# R. L. Stevenson -> Thomas -> Robert -> Thomas Smith
# David Alan -> David -> Robert -> Thomas Smith
# Katharine -> Alan -> Robert -> Thomas Smith
```

The first block gives the lifespans (78, 58, 71 and 69 years) and the lighthouse rates. Alan's 13 lights in the 11 years from 1843 to 1853 is 1.18 a year; David's 28 listed lights from 1854 to 1880 is 1.04; Thomas's 31 from 1854 to 1886 is 0.94. These are crude: David and Thomas designed many lights together, so the same lighthouse appears on both lists and the counts must not be added; the pages differ over their own totals (Alan's page says 13 but lists 14 entries, counting Hoy High and Hoy Low as one; David's lead says "over 30" but lists 28); and Robert's page lists about 22 lights, but the extract I was given did not show the full span, so I left his rate out. Robert's 1797 to 1842 is 45 years, though the page calls it "nearly fifty", a conflict inside one page. Alan's term as Engineer in Chief begins in 1843, so there is no gap after Robert's. From Robert's appointment in 1797 to Thomas's last lighthouse (Ailsa Craig, 1886) is 89 years. Whether the family held the engineering post for all of those years is not something I can say: the pages give Robert's and Alan's years, but only say that David moved to the Board in 1853 and that Thomas designed from 1854, without giving the length of either man's service. The last three lines show the family tree: Robert Louis Stevenson, David Alan and Katharine all trace to Robert and, through him, to Smith.

	ROBERT	ALAN	DAVID	THOMAS
Lived	8 Jun 1772 to 12 Jul 1850	28 Apr 1807 to 23 Dec 1865	11 Jan 1815 to 17 Jul 1886	22 Jul 1818 to 8 May 1887
With the Board	engineer 1797 to 1842	Engineer in Chief 1843 to 1853	moved to the Board 1853	no dates given for the post
Best known for	Bell Rock (1811); Fresnel lenses and flashing systems	Skerryvore; the 1848 account	lights with Thomas 1854 to 1880, including Muckle Flugga; help with lights in Japan	31 lights, 1854 to 1886; the Stevenson screen; the dynamometer

	ROBERT	ALAN	DAVID	THOMAS
Children named on the page	Alan, David, Thomas, Jane (1801 to 1864, her father's secretary)	three daughters and a son	Charles Alexander and David Alan (the page says three children, names two)	Robert Louis Stevenson
Training	under his stepfather Thomas Smith	University of Edinburgh from 1821 (for the church), then an apprenticeship from 1823 in the family business	Edinburgh High School and the University; partner in R & A Stevenson in 1838	not given on the page

What Each of Them Did

Robert Stevenson's page credits him with the movable jib and balance crane, with the use of Fresnel lenses (see How Lighthouses Work 4), with reflector design, and with rotating and shuttering systems that gave lights their distinct characters (How Lighthouses Work 5); for the last he received a gold medal from William I of the Netherlands. He was elected a Fellow of the Royal Society of Edinburgh in 1815. His page records the Bell Rock dispute that How Lighthouses Work 6 mentioned: John Rennie was a consulting engineer and Samuel Smiles's account favoured Rennie, but the Board credited Stevenson, and the page says historians have followed the Board. I give the dispute as the page states it and have not looked at the underlying sources.

Alan, the eldest son, was elected a Fellow of the Royal Society of Edinburgh in 1838 and was given an honorary law degree by the University of Glasgow in 1840. His page says he was Engineer in Chief from 1843 to 1853 and built 13 lighthouses in that time; the Skerryvore page adds that Thomas supervised the interior fitting out in 1843. Neither page gives a reason for Alan leaving the post in 1853, and his own page says nothing about lenses; that is a gap in the page, not evidence that he did no optical work. David joined the Board in 1853 and, with Thomas, designed lighthouses from 1854 to 1880; his page also says he helped Richard Henry Brunton with lighthouses in Japan, including a method to withstand earthquakes, and that his son David Alan built nearly thirty more.

Thomas is the one whose range goes beyond lighthouses. His page calls him a civil engineer, lighthouse designer and meteorologist. He co-founded the Scottish Meteorological Society,

devised the Stevenson screen for sheltering weather instruments (the page gives no date), published a wave-height paper in 1852 and *The Design and Construction of Harbours* in 1864, and in 1869 ran an electric light experiment at Granton Harbour, with a light on the Trinity Chain Pier controlled from half a mile away. He was president of the Royal Society of Edinburgh from 1884 to 1886. Harbours were not always kind to him: construction of the Wick breakwater began in April 1863 and, according to the page, was abandoned in 1877 after storm damage.

Measuring the Sea: the Wave Dynamometer

In How Lighthouses Work 6 the Skerryvore page's figure of 6,000 lb per square foot raised a question: where did a number like that come from, and does the sea really push that hard? The answer is an instrument. Alan's 1848 account says that Thomas had "lately contrived an instrument for the purpose" of registering the force of wave impulses and had applied it at various parts of the coast; the reading tool reported that the account gives his paper in the *Transactions of the Royal Society of Edinburgh* as 20 January 1845. The Trinity House article calls it the marine dynamometer, built to record impact pressures, deployed on the Bell Rock and Skerryvore, and notes that in 1848 Alan wrote that no systematic attempt had been made to measure the force of a wave. The same article calls such measurements rare. It does not describe the instrument's mechanism, and neither did the part of the account I read, so I will not describe one. A search summary said Thomas collected the first systematic wave-impact measurements over some 23 months with three of the instruments, but I did not find that in the pages I read, so treat it as unconfirmed.

The readings I did find, all in pounds per square foot, are in the account: a best result at Skerryvore of 4,335 "as yet", a Bell Rock best of 3,013 (described as "one-fourth part" less), and average readings of 611 for the summer months and 2,086 for the winter months of 1843 and 1844. The account says the experiments "have not been continued long enough" to be fully useful. The Skerryvore page says the readings implied the tower had to resist 6,000 pounds per square foot (about 300 kPa), which is more than the 4,335 in the 1848 text. A search result quoted a peak of 6,083 from a travel-company article that I could not check. So I have three possible numbers for the Skerryvore peak (4,335, 6,000 and 6,083) and I do not know whether the larger ones come from later readings or from a design allowance; the sources I read do not say. The code converts them all.

```
  # Thomas Stevenson's marine dynamometer readings, as quoted in Alan Stevenson's 1848 acc
```

```
LBF_N = 4.4482216
FT_M = 0.3048
RHO_SEA, G = 1025.0, 9.81          # assumed seawater density, gravity

def kpa(lb_per_ft2):
    return lb_per_ft2 * LBF_N / FT_M ** 2 / 1000

READINGS = (                       # lb per square foot
    ("summer mean 1843-44", 611),
    ("winter mean 1843-44", 2086),
    ("Bell Rock best", 3013),
    ("Skerryvore best (1848)", 4335),
    ("design figure on the Skerryvore page", 6000),
)
for label, value in READINGS:
    head = kpa(value) * 1000 / (RHO_SEA * G)
    print(label.ljust(38), value, "lb/ft2 =", round(kpa(value), 1), "kPa = head", round(

print("winter over summer:", round(2086 / 611, 2))
print("Bell Rock below Skerryvore:", round((1 - 3013 / 4335) * 100, 1), "per cent (the t
print("design over best reading:", round(6000 / 4335, 2))
print("design over winter mean:", round(6000 / 2086, 2))
print("a reading of 6083 lb/ft2:", round(kpa(6083), 1), "kPa")
# summer mean 1843-44                  611 lb/ft2 = 29.3 kPa = head 2.9 m of seawater
# winter mean 1843-44                  2086 lb/ft2 = 99.9 kPa = head 9.9 m of seawater
# Bell Rock best                       3013 lb/ft2 = 144.3 kPa = head 14.3 m of seawat
# Skerryvore best (1848)               4335 lb/ft2 = 207.6 kPa = head 20.6 m of seawat
# design figure on the Skerryvore page   6000 lb/ft2 = 287.3 kPa = head 28.6 m of seawat
# winter over summer: 3.41
# Bell Rock below Skerryvore: 30.5 per cent (the text says one-fourth)
# design over best reading: 1.38
# design over winter mean: 2.88
# a reading of 6083 lb/ft2: 291.3 kPa
```

The 6,000 figure is 287.3 kPa, the same as How Lighthouses Work 6 found, and corresponds to a

the Bell Rock reading is 144.3 kPa, and the winter average of 2,086 is about 99.9 kPa, close to one atmosphere (101.3 kPa), while the summer average of 611 is only 29.3 kPa. So the winter average was 3.41 times the summer one. Two checks on the text: 3,013 is 30.5 per cent below 4,335, not the 25 per cent that "one-fourth less" suggests (a quarter off 4,335 would be about 3,251), which may be rounding, a different pair of readings or a misprint; I cannot tell which. And

the 6,000 figure is 1.38 times the best 1848 reading, which would be a sensible margin if it was a design allowance (my reading, not stated), and 2.88 times the winter average. If 6,083 is a later reading, it exceeds 6,000 by only 1.38 per cent, so the two are consistent with one another.

SOURCE	SKERRYVORE FIGURE	WHAT IT SAYS
Alan Stevenson, 1848	4,335 lb per square foot	"the greatest result as yet obtained"; Bell Rock 3,013; seasonal means 611 and 2,086
Skerryvore page	6,000 lb per square foot (about 300 kPa)	readings implied the tower had to resist this
A travel-company article (via search)	6,083 lb	unchecked; not in the pages I read
Trinity House article	none quoted	the instrument recorded impact pressures; such data are rare

Waves and the Wave-Height Rule

Thomas's 1852 paper concerned how big waves get. His page gives the rule H = 1.5 times the square root of F, where H is the wave height in feet and F the fetch (the distance of open water the wind blows over) in miles, and says his observations were for fetches under 100 kilometres. Elsewhere the same page says "around 100 metres", which looks like a slip for kilometres (my reading). I give the rule as the page does; the full published formula may include other terms, and I did not check it. The code tries the rule at several fetches and runs it backwards: how long a fetch would give an 8 or 10 m wave, which is the size the page quotes for the Wick estimate? I did not confirm how that estimate was reached.

```
# Stevenson's wave-height rule as the Thomas Stevenson page gives it: H = 1.5 * sqrt(F),
import math

FT_M = 0.3048
MILE_KM = 1.609344

def wave_height_ft(fetch_miles):
    return 1.5 * math.sqrt(fetch_miles)

def fetch_for_height_miles(height_ft):
    return (height_ft / 1.5) ** 2
```

```
for fetch in (1, 10, 50, 100):
    h = wave_height_ft(fetch)
    print("fetch", fetch, "miles =", round(fetch * MILE_KM, 1), "km -> wave", round(h, 1
print("fetch of 100 km =", round(100 / MILE_KM, 1), "miles -> wave", round(wave_height_f
for metres in (8, 10):
    feet = metres / FT_M
    miles = fetch_for_height_miles(feet)
    print(metres, "m =", round(feet, 1), "ft needs a fetch of", round(miles), "miles =",
for fetch in (4, 25, 400):          # doubling the height needs four times the fetch
    print("fetch", fetch, "-> doubled fetch ratio of height:", round(wave_height_ft(4 *
# fetch 1 miles = 1.6 km -> wave 1.5 ft = 0.5 m
# fetch 10 miles = 16.1 km -> wave 4.7 ft = 1.4 m
# fetch 50 miles = 80.5 km -> wave 10.6 ft = 3.2 m
# fetch 100 miles = 160.9 km -> wave 15.0 ft = 4.6 m
# fetch of 100 km = 62.1 miles -> wave 11.8 ft
# 8 m = 26.2 ft needs a fetch of 306 miles = 493 km
# 10 m = 32.8 ft needs a fetch of 478 miles = 770 km
# fetch 4 -> doubled fetch ratio of height: 2.0
# fetch 25 -> doubled fetch ratio of height: 2.0
# fetch 400 -> doubled fetch ratio of height: 2.0
```

A fetch of 1 mile gives 1.5 ft (0.5 m), 10 miles 4.7 ft, 50 miles 10.6 ft and 100 miles 15 ft (4.6 m). Because of the square root, quadrupling the fetch only doubles the height; the last lines show that, with a ratio of 2.0 each time. Inside the range of the observations, 100 km (62.1 miles) gives 11.8 ft, about 3.6 m. To reach 8 m the rule needs a fetch of 306 miles (493 km), and 10 m needs 478 miles (770 km), far outside the 100 km. So either the Wick estimate came from a different method, or it extends the rule beyond what it was fitted to; I do not know which, and I would not trust the rule that far. Note also that this rule is about the height of the waves, not the impact pressure of a breaking wave on a tower, which is what the dynamometer was built to record.

The Family Link to Robert Louis Stevenson

The most famous Stevenson is not an engineer. Robert Louis Stevenson was Thomas's son and Robert's grandson, and his page names Alan and David as his uncles. He named his Bournemouth house Skerryvore after the lighthouse that his uncle Alan built. The next table,

with ages worked out from the page's remark that he qualified for the bar in July 1875, aged 24, shows how little time he spent in the family trade.

DATE	WHAT THE PAGE SAYS	AGE (MY ARITHMETIC)
Nov 1867	enters the University of Edinburgh to study engineering, showing little enthusiasm	about 16
1868	about six weeks at Wick, where the family was building a sea wall	about 17
1869	goes with his father on an official tour of the Orkney and Shetland lights	about 18
1870	three weeks on Erraid	about 19
Apr 1871	tells his father he will be a writer; the compromise is law	about 20
Jul 1875	qualifies for the Scottish bar; never practises	24

His page says he got more material for writing from the lighthouse trips than interest in the engineering, that his father was disappointed, and that his mother said he was "wonderfully resigned". In 1887, the year his father died, his collection *Underwoods* reflects on turning away from the family profession. Exercise 1 works out the ages. The page does not mention the wave dynamometer, and I found no source that links the writer's work to the instrument, so I will not suggest a link.

Where the Pages Disagree

The family is well documented, but the pages I read contradict themselves in several places. These are the ones I noticed: the Board's formation (an Act of 1786, against 1867 in its infobox); Robert's service (1797 to 1842 is 45 years, called "nearly fifty"); Bell Rock built 1807 to 1810 but listed as 1811, the year it was first lit; Dubh Artach listed on Robert's page as a design of 1872, 22 years after his death; the number of Alan's lighthouses (13 against 14 entries) and David's ("over 30" against 28); a son of Alan dated 1827 although the marriage was in 1844; the height of Muckle Flugga; and the Skerryvore peak pressure (4,335, 6,000 or 6,083). I have no way to resolve these from the pages alone.

Common mistakes

Adding the lighthouse counts of David and Thomas as if they were separate (many were joint); treating the Board's formation year as 1867; mixing up which Stevenson built which tower; treating 6,000 lb per square foot as a measured reading rather than a figure on one page that is higher than the 1848

account's 4,335; saying the Bell Rock reading is "a quarter lower" when the numbers give 30.5 per cent; using the wave-height rule for fetches far beyond 100 km; quoting a figure I found only through a search result as if it were in the sources; and calling Robert Louis Stevenson an engineer because the family was.

EXERCISE 1

Build the family timeline. Using the four lifespans, find how many of the four were alive in 1800, 1820, 1840, 1860, 1880 and 1890. Then find each man's age at the events the pages give (Alan elected FRSE in 1838 and made Engineer in Chief in 1843, David moving to the Board in 1853, Thomas's first lighthouse in 1854, Robert's appointment in 1797), and Robert Louis Stevenson's approximate age at each of the dates in the table, taking him as 24 in 1875.

View solution

EXERCISE 2

Treat the five pressures (611, 2,086, 3,013, 4,335 and 6,000 lb per square foot) as acting on a disc 3 inches across (an assumed size, for practice only). Find the area of the disc, the force on it for each pressure in pounds-force and newtons, and each pressure as a percentage of 6,000. Then test the claim that the Bell Rock figure is one-fourth below the Skerryvore one, and compare 6,083 with 6,000.

View solution

EXERCISE 3

Write the wave-height rule for fetches in kilometres and heights in metres. Find the height for fetches of 5, 20, 50, 100 and 200 km, marking the ones beyond the 100 km of the observations. Then run it backwards to find the fetch needed for waves of 2, 4, 6, 8 and 10 m, and check that doubling the height needs four times the fetch.

View solution

QUICK REFERENCE

- **Robert (1772 to 1850):** Board engineer 1797 to 1842; trained under stepfather Thomas Smith, who joined the Board in 1786; Bell Rock; credited with movable jib and balance crane, Fresnel lenses and

flashing systems

- **Alan (1807 to 1865):** Engineer in Chief 1843 to 1853; 13 lights; Skerryvore; *Account of the Skerryvore Lighthouse* (1848)
- **David (1815 to 1886):** moved to the Board 1853; lights with Thomas 1854 to 1880 incl. Muckle Flugga; son David Alan continued the work
- **Thomas (1818 to 1887):** 31 lights, 1854 to 1886; Stevenson screen; wave-height paper 1852; father of Robert Louis Stevenson
- **Northern Lighthouse Board:** Erection of Lighthouses Act 1786 (royal assent 27 June); 208 statutory lighthouses at 31 March 2024; George Street, Edinburgh
- **Dynamometer readings (lb per sq ft):** summer mean 611, winter mean 2,086, Bell Rock best 3,013, Skerryvore best 4,335 (1848); 6,000 on the Skerryvore page (287.3 kPa); 6,083 unchecked
- **Wave rule (as the page gives it):** H = 1.5 times the square root of F, H in feet, F in miles; observations for fetches under 100 km
- **Conflicts:** Board founded 1786 or 1867; Robert's service 45 years or "nearly fifty"; Alan's lights 13 or 14; David's "over 30" or 28; Skerryvore peak 4,335, 6,000 or 6,083; Bell Rock "one-fourth" below Skerryvore or 30.5 per cent

COMING NEXT

How Lighthouses Work 8 is about fog and radio: fog bells, horns and sirens, sound in air, Racon and radar, lightvessels, automation and the end of keepers.

How Lighthouses Work

Chapter 8 · Fog and Radio

Everything in How Lighthouses Work 1 to 7 assumed that the sailor could see the light. Fog takes that away. A lamp with a range of 20 nautical miles is no use when the visibility is a few hundred metres, and the rocks that the Stevensons built on do not go away because the weather has closed in. This chapter is about what lighthouse authorities did instead: they made noise, then they used radio, and in the end they took the people out of the towers altogether. We will look at fog bells, guns, horns and sirens, at how sound behaves in air, at the radar beacon called a racon, at lightvessels, and at the automation that ended the keepers.

WHAT WAS RUN, AND WHAT TO TRUST

Every Python snippet in this chapter and in the three exercise solutions was run, and the output in the comments is what Python printed. The facts come from the English Wikipedia text for the pages on foghorns (fog signals), fog bells, sirens, racons, lightvessels, lighthouses, lighthouse keepers, the Northern Lighthouse Board, Skerryvore, Bell Rock, Muckle Flugga and the speed of sound. I read only parts of the longer pages, mostly by searching them for the words I needed, so a statement that a page does not say something means only that I did not see it. I could not check any figure against the printed sources those pages cite. Where two pages, or two parts of one page, disagree, the chapter says so. The *assumptions* are mine and are labelled: the gas constant and the molar mass of dry air used in the sound-speed model, and the dates that are given as a year only. All sums, conversions and ratios are my arithmetic. None of this is a navigation aid: it is a set of worked examples.

Noise When the Light Fails

The foghorn page opens with the job: a device that uses sound to warn vehicles of navigational hazards in fog, for when "visual navigation aids such as lighthouses are obscured". Its history section says audible fog signals have been used "for hundreds of years", at first seashell horns, fog bells or gongs struck by hand. The fog bell page dates the first bell made for the purpose to 1766 at Nidingen on the Baltic Sea. Bells were rung by hand, which was a chore in a long fog, or worked by clockwork that had to be wound, as in the clockwork of How Lighthouses Work 5. An American, Andrew Morse Jr., made a "perpetual fog bell" used in 1839 at Penobscot Bay, driven by a long beam that rose and fell with the waves; the page adds that the float was quickly carried out to sea. The same page says there were 93 fog bells in the United States in 1878, and

that bells had a short range, especially in wind. In England most mechanical fog bells were taken out of use around 1905.

Some places used guns. The foghorn page says a small cannon was fired at some lighthouses, but it was labour-intensive and dangerous, and that Trinity House, the English authority, did not use steam whistles and preferred an explosive signal. It later developed a system for firing a gun-cotton charge electrically, though the charge had to be replaced by hand each time. At Portland Bill, with a signal every five minutes, that meant lowering the horns, inserting two new charges and raising them again every five minutes in fog. In Scotland the Skerryvore page says a fog bell had been installed by the time of the keeper James Tomison, who joined in 1861, and calls it one of only two operating in Scotland at the time. The Bell Rock page records a much older legend: that a warning bell was fixed on the rock in the 14th century by the Abbot of Arbroath and stolen a year later by a pirate, a story told in Robert Southey's poem. I give it as legend because the page does.

Then the mechanics improved. The foghorn page says the first automated steam-powered foghorn was invented by Robert Foulis, a Scot who emigrated to Saint John, New Brunswick, and was built at Partridge Island in 1859 (as the "Vernon-Smith horn", because the plans were passed on under another man's name; Foulis was later credited). He is said to have noticed on a foggy night that low notes carried better than high ones, so the device made a low note and a coded pattern. The page adds that a Captain Newton also claimed the idea of loud, low-pitched signals, so the credit is disputed. In the United States Celadon Daboll built a coal-powered trumpet, which was not universally adopted. In Britain John Tyndall and Lord Rayleigh ran experiments, and Rayleigh's work for Trinity House led to a siren with a large trumpet installed at Trevose Head in 1913. The siren page gives an earlier line: sirens with slotted cylinders were developed in 1877 to 1880 by James Douglass and George Slight of Trinity House, and the final version was first installed in 1887 at the Ailsa Craig lighthouse. Ailsa Craig was also where Thomas Stevenson's last lighthouse (1886) was, in How Lighthouses Work 7, but the pages do not say the two installations are the same, so I only note the shared name. From the early twentieth century, the foghorn page says, the diaphone, an organ stop invented by Robert Hope-Jones and developed as a fog signal by John Northey of Toronto, became the standard apparatus for new installations; it ran on compressed air and made very powerful low notes.

SIGNAL	HOW IT SOUNDS	WHAT THE PAGES SAY
Fog bell	struck by hand, clockwork or the waves	first made for the purpose in 1766; short range, worse in wind; English mechanical bells out of use around 1905

SIGNAL	HOW IT SOUNDS	WHAT THE PAGES SAY
Gun or explosive	a charge fired at set intervals	labour-intensive and dangerous; Trinity House used gun-cotton charges that had to be replaced by hand
Daboll trumpet	a vibrating plate or reed, coal-powered	American; not universally adopted; a few lasted to the mid-twentieth century
Siren	air forced through slots in rotating cylinders	Trinity House development 1877 to 1880; Ailsa Craig 1887; large-trumpet siren at Trevose Head 1913
Diaphone	compressed air; very powerful low notes	from an organ stop; became the standard for new installations in the early twentieth century
Electric horn	diaphragm or compressed air	replaced older horns as lights were automated in the 1960s and 1970s; switched on by a beam that is reflected back by fog

Sound in Air

Light covers 20 miles in a fraction of a millisecond. Sound is a different thing altogether. The speed of sound page says that at 20 °C it is about 343 m/s in air, about 331 m/s in dry air at 0 °C, and 1,481 m/s in fresh water at 20 °C, a little over four times faster. In still air it depends mostly on temperature, and the page adds that humidity raises it by 0.1% to 0.6%. History is on the page too. Newton's 1687 figure of 979 ft/s is described as too low by about 15%, an error the page ascribes to treating the compression of air as isothermal instead of adiabatic, and the William Derham who in 1709 timed gunshots from a church tower wrongly concluded that rain and fog slow sound down, a conclusion accepted until Tyndall disproved it. That last point matters here: fog does not slow the signal. What the page does describe is refraction: because temperature falls with height, sound is bent upward, away from listeners on the ground, which leaves an acoustic shadow at some distance from the source.

The ideal-gas formula for the speed of sound is the square root of (adiabatic index times gas constant times absolute temperature, divided by molar mass). The page gives the adiabatic index as about 1.4 for air. I supplied the gas constant and the molar mass of dry air from memory, so they are my assumptions. The code runs the model for five temperatures and then uses it to answer two practical questions: how long a signal takes to reach a ship, and how far apart a sound travelling through the air and one travelling through the water would arrive.

```python
# Speed of sound in dry air from an ideal-gas model, and what that means for a fog signa
import math

GAMMA = 1.4          # adiabatic index of air (the page gives "about 1.4")
R = 8.314462618      # gas constant, J/(mol K) (my assumed standard value)
M_AIR = 0.0289645    # molar mass of dry air, kg/mol (my assumed standard value)
V_WATER = 1481       # m/s, fresh water at 20 C, from the page

def v_air(celsius):
    return math.sqrt(GAMMA * R * (celsius + 273.15) / M_AIR)

for t in (-10, 0, 10, 20, 30):
    print(t, "C:", round(v_air(t), 1), "m/s")

print("water / air at 20 C:", round(V_WATER / v_air(20), 2))
for km in (1, 2, 5, 10):
    print(km, "km at 10 C:", round(km * 1000 / v_air(10), 1), "s")
d = 1000
print("1 km, air minus water at 20 C:", round(d / v_air(20) - d / V_WATER, 2), "s")
# -10 C: 325.2 m/s
# 0 C: 331.3 m/s
# 10 C: 337.3 m/s
# 20 C: 343.2 m/s
# 30 C: 349.0 m/s
# water / air at 20 C: 4.31
# 1 km at 10 C: 3.0 s
# 2 km at 10 C: 5.9 s
# 5 km at 10 C: 14.8 s
# 10 km at 10 C: 29.6 s
# 1 km, air minus water at 20 C: 2.24 s
```

Exercise 2 puts Newton's 979 ft/s (298 m/s) 13.1% below the model's 20 °C value, where the page says about 15%; the page does not say what temperature that figure refers to, so I cannot say whether the difference matters. The model gives 331.3 m/s at 0 °C (the page says about 331) and 343.2 m/s at 20 °C (the page says about 343), so it agrees with the two values I could check. The ratio of the water speed to the air speed at 20 °C is 4.31, matching "a little over four times". A signal 5 km away takes about 14.8 s at 10 °C, and a ship 10 km away hears the horn almost 30 s after it started. The foghorn page says each horn was given its own timing characteristics, set by a clockwork coder, so that mariners could identify it, much as lights have their characters.

Over 1 km at 20 °C, a sound in water would arrive 2.24 s before the same sound in air. That is the idea behind the submarine bells I describe below: a ship that can listen to both gets a time difference to turn into a distance. The page on fog bells says the bells were later replaced by electric transducers, and that the underwater work led, through Reginald Fessenden, to sonar. I have not checked that history beyond the page.

Lightvessels

Where a rock or sandbank is too deep or too soft for a tower, the light goes on a ship. The lightvessel page opens by saying the first modern lightship was invented by Robert Hamblin in 1734 and was off the Nore sandbank in the Thames estuary. Its section on British lightships says David Avery and Robert Hamblin put the earliest one there in 1731, a private venture that made a profit without the government enforcing payment. That is a conflict inside one page (1734 or 1731), and I cannot resolve it from the page. Other vessels followed off Norfolk (1736), Owers Bank (1788) and the Goodwin Sands (1793), and Trinity House later displaced the private operators. The first United States lightship was established in Chesapeake Bay in 1820.

The Scottish connection is here too. A converted fishing boat renamed *Pharos*, meaning lighthouse, entered service on 15 September 1807 "near Inchcape, Scotland". The Bell Rock page says that Bell Rock is also known as Inchcape, so I take this to be the rock Robert Stevenson was building on. That would put a lightvessel on station four years before the tower's light of 1811, although the pages do not say that the vessel was there for the building work. Its anchor, the first of the mushroom type on a lightvessel, weighed 1.5 tons, and the page gives a rule of thumb for the chain: 6 feet for every foot of water. Cast-iron chain from the 1820s made the mooring far more effective. Lightvessels also carried bells. In the early twentieth century, tests by Trinity House found that a bell lowered about 18 feet could be heard at 15 miles, but that the practical range in operating conditions was 1 to 3 miles. The code below checks the list of Trinity House hulls on the page against the count the same page states, and works out the chain and the bell ranges.

```
# Lightvessels: the hull-number list against the stated count, the mooring rule and the
SOLAR = (2, 5, 6, 7, 9, 10, 17)
CLASS_20 = (19, 22, 23, 25)
FLOATS = ("LF2", "LF3")
print("hulls listed:", len(SOLAR) + len(CLASS_20), "lightvessels and", len(FLOATS), "flo
print("the page's own count: 8 lightvessels and 2 floats")
```

```
for depth_m in (10, 20, 30):
    print(depth_m, "m of water ->", 6 * depth_m, "m of chain (6 ft of chain per foot)")

FT, MILE = 0.3048, 1609.344
print("bell depth 18 ft =", round(18 * FT, 1), "m; heard at 15 miles =", round(15 * MILE
for miles in (1, 3):
    print("practical", miles, "mile =", round(miles * MILE / 1000, 1), "km =",
          round(100 * miles / 15), "% of the test range")
print("years from the Pharos lightvessel (1807) to Bell Rock's light (1811):", 1811 - 18(
print("years from 1731 to 1734:", 1734 - 1731)
# hulls listed: 11 lightvessels and 2 floats
# the page's own count: 8 lightvessels and 2 floats
# 10 m of water -> 60 m of chain (6 ft of chain per foot)
# 20 m of water -> 120 m of chain (6 ft of chain per foot)
# 30 m of water -> 180 m of chain (6 ft of chain per foot)
# bell depth 18 ft = 5.5 m; heard at 15 miles = 24.1 km
# practical 1 mile = 1.6 km = 7 % of the test range
# practical 3 mile = 4.8 km = 20 % of the test range
# years from the Pharos lightvessel (1807) to Bell Rock's light (1811): 4
# years from 1731 to 1734: 3
```

The page lists seven solar lightvessels (hull numbers 2, 5, 6, 7, 9, 10 and 17) and four of the larger "20 class" (19, 22, 23 and 25), which is 11 lightvessels, plus two light floats (LF2 and LF3). The page's own text says there are "currently eight unmanned lightvessels and two smaller light floats". The two counts disagree and I cannot tell which is out of date; the page does not say whether some of the numbered hulls are spare or retired. The chain rule gives 60 m of chain for 7% to 20% of the 15-mile test range, which is a reminder that a figure from a trial in good conditions says little about a bad night. The page also says the crews of lightvessels kept records of passing ships, observed the weather and sometimes carried out rescues, and that the 20 class, which relies on diesel-electric generators, is used where a main light with a range beyond 20 nautical miles is needed, because a solar lightvessel's main light reaches at most 19.

Radar and the Racon

Radar made a new kind of fog signal possible. The racon, or radar beacon, is defined by the International Telecommunication Union as a transmitter-receiver on a fixed navigational mark

which, when triggered by a radar, automatically returns a distinctive signal that appears on the radar display and gives range, bearing and identification. The racon page says it answers on the same frequency with a Morse character, drawn as a short line of dots and dashes radiating away from the beacon on the screen, usually a few nautical miles long. Racons mostly work in the X band (9320 to 9500 MHz) and many also in the S band (2920 to 3100 MHz). A modern one is frequency-agile: it detects the incoming pulse, tunes its transmitter and replies with a 25 microsecond signal within 700 nanoseconds. Older ones sweep slowly across the band over a minute or two and answer only about 5% of the time. The page says the large majority are on buoys rather than lighthouses, and that they mark things such as bridge spans and offshore platforms. The foghorn page reports an older version of the idea: underwater fog bells ringing a coded number such as 23, which it calls "an earlier precursor to RACON".

A racon must not hide real targets behind it, so it is silent for part of the time. The page gives the UK duty cycle as about 30%, usually 20 s on and 40 s off, or sometimes 9 s on and 21 s off (as on the Sevenstones lightship). The code turns the speed of light into timing, to show what these microsecond numbers mean on the screen. I use the vacuum speed of light, which is very slightly above the speed in air, so the numbers are close but not exact.

```
# Radar timing: how long a pulse takes to go out and come back, and what a racon adds.
C = 299_792_458      # m/s, speed of light in vacuum (radar in air is very slightly slow
NM = 1852            # metres in a nautical mile

def echo_us(range_nm):
    return 2 * range_nm * NM / C * 1e6

for nm in (1, 3, 6, 12, 24):
    print(nm, "nm: echo returns after", round(echo_us(nm), 1), "microseconds")

reply_us = 25                        # racon reply length, from the page
print("25 us of reply, as range:", round(reply_us * 1e-6 * C / 2), "m =", round(reply_us
print("700 ns of response time, as range:", round(700e-9 * C / 2), "m")

for label, on, off in (("UK, 20 s on / 40 s off", 20, 40), ("UK, 9 s on / 21 s off", 9, 
                       ("US buoy, 20 on / 20 off", 20, 20)):
    print(label, "-> responds", round(100 * on / (on + off), 1), "% of the time")
# 1 nm: echo returns after 12.4 microseconds
# 3 nm: echo returns after 37.1 microseconds
# 6 nm: echo returns after 74.1 microseconds
# 12 nm: echo returns after 148.3 microseconds
# 24 nm: echo returns after 296.5 microseconds
```

```
  # 25 us of reply, as range: 3747 m = 2.02 nm
  # 700 ns of response time, as range: 105 m
  # UK, 20 s on / 40 s off -> responds 33.3 % of the time
  # UK, 9 s on / 21 s off -> responds 30.0 % of the time
  # US buoy, 20 on / 20 off -> responds 50.0 % of the time
```

One nautical mile of range costs 12.4 microseconds of echo delay, there and back, so a target at 12 nm returns after 148.3 microseconds. A reply that lasts 25 microseconds therefore smears over about 3,747 m, or 2.02 nm, of range on the display, which fits the page's "few nautical miles". The 700 ns response time is worth about 105 m, so even a quick racon puts its mark slightly beyond the true position. The duty cycles are 33.3% for 20 s on, 40 s off, a little over the page's "about 30%", and exactly 30.0% for 9 s on, 21 s off; the American battery buoy cycle of 20 s on, 20 s off is 50%. I did not check the 33.3% against the page's "about"; it is simply the arithmetic of the figures.

	LIGHT	SOUND	RADAR AND RACON
Carried by	visible light	pressure waves in air (or water)	radio pulses
Speed	about 299,792,458 m/s in vacuum	about 343 m/s in air at 20 °C	the same as light
Trouble in fog	the range falls with the visibility	no slower; bent upward by temperature gradients, leaving shadows	does not depend on seeing the tower; needs a radar on board
Tells the	a flash pattern and a	a direction, roughly; a rough	range, bearing and an

Automation and the End of the Keepers

The foghorn page says that once automation became common in the 1960s and 1970s most older foghorns were removed rather than keep their complex machinery running, and replaced by electric diaphragm or compressed-air horns. Activation is automatic: a laser or photo beam is sent out to sea, and if it is reflected back because of fog or rain, a sensor starts the horn. The page adds that modern aids including GPS have in many cases made large long-range foghorns unnecessary, according to the International Association of Lighthouse Authorities. The

lighthouse page makes the same point for the keepers: electrification and automatic lamp changers "began to make lighthouse keepers obsolete". It says that for years lighthouses were still staffed partly because keepers could serve as a rescue service, and that GPS and better navigation brought the phasing out of staffed lights. The lighthouse keeper page lists the other pieces: remote monitoring and automatic bulb changing from the 1960s, and it says that keepers still working today do building maintenance and repair buoys and technical equipment. The lighthouse page also describes how an automated light often replaces the rotating lens (How Lighthouses Work 4 and 5) with a high-intensity flasher that concentrates the light in time rather than direction.

For Scotland the pages give these dates. The Bell Rock page says the light has been automated since 24 October 1988 and that the Northern Lighthouse Board, which has been at 84 George Street in Edinburgh since 1832, monitors it remotely. The Skerryvore page says the lighthouse was automated in 1994, and the Muckle Flugga page says that the lighthouse was automated in 1995, when the last residents moved out. The lighthouse keeper page says that Britain's automation began in the late 1960s but that most lights kept their keepers until the 1980s and 1990s, and that the keepers left the Fair Isle South lighthouse on 31 March 1998, when the last staffed Scottish light became automated. The Northern Lighthouse Board page says it maintained foghorns at a number of locations between 1876 and 2005 and that the last, at Skerryvore, sounded for the final time on 4 October 2005. The two dates are for different things, so they do not conflict: the keepers left Skerryvore in 1994, 11 years before its horn was silenced. Why the horn was kept going so long is something the pages do not explain, so I will not guess.

```
# Three Scottish rock lights, from first lit to automation, and the last foghorn.
from datetime import date

LIGHTS = (("Bell Rock", 1811, 1988), ("Skerryvore", 1844, 1994), ("Muckle Flugga", 1858,
for name, lit, auto in LIGHTS:
    print(name, "| lit", lit, "| automated", auto, "| years between:", auto - lit)

LAST_KEEPERS = date(1998, 3, 31)    # Fair Isle South, the last staffed Scottish light
LAST_HORN = date(2005, 10, 4)       # Skerryvore, the last NLB foghorn
print("days from last keepers to last horn:", (LAST_HORN - LAST_KEEPERS).days)
print("years of NLB foghorns, 1876 to 2005:", 2005 - 1876)
print("years from the 1786 Act to the last keepers:", LAST_KEEPERS.year - 1786)
print("Skerryvore: keepers left", 1994, "- horn sounded for another", LAST_HORN.year - 1
# Bell Rock | lit 1811 | automated 1988 | years between: 177
# Skerryvore | lit 1844 | automated 1994 | years between: 150
# Muckle Flugga | lit 1858 | automated 1995 | years between: 137
```

```
# days from last keepers to last horn: 2744
# years of NLB foghorns, 1876 to 2005: 129
# years from the 1786 Act to the last keepers: 212
# Skerryvore: keepers left 1994 - horn sounded for another 11 years
```

From first lighting to automation the three towers served for 177 years (Bell Rock, 1811 to 1988), 150 years (Skerryvore, 1844 to 1994) and 137 years (Muckle Flugga, 1858 to 1995). These are span lengths, not records of continuous staffing: Skerryvore, for instance, was dark for five years after a fire in 1954 (that is from the Skerryvore page). The period from the Board's act of 1786 to the last Scottish keepers in 1998 is 212 years, and the last foghorn sounded 2,744 days, a little over seven years and six months, after the last keepers left. Nothing here tells us anything about the people who lived in those years, which is a different story from the dates.

LIGHT	FIRST LIT	AUTOMATED	YEARS BETWEEN
Bell Rock	1811 (1 Feb)	1988 (24 Oct)	177
Skerryvore	1844	1994	150
Flugga	1854)		
Fair Isle South	not looked up	1998 (31 Mar), the last staffed Scottish light	not worked out

Common mistakes

Saying fog slows sound (Derham thought so, and Tyndall showed it did not); using 343 m/s as if it were true in all weather (it is about 3.5% too far in distance at 0 °C by my arithmetic); mixing up the time for sound in water and in air; forgetting that a racon is silent for part of every cycle; taking the radar range as one-way when the echo has to go there and back; dating the first lightship to 1734 and 1731 without saying that the page gives both; treating the practical bell range of 1 to 3 miles as the same as the 15-mile test; counting 8 and 11 Trinity House lightvessels as the same list; confusing the year the keepers left a tower with the year its horn was silenced; and assuming that every Scottish light lost its keepers in the same year.

EXERCISE 1

Build the automation timeline. Put the five dated events (Bell Rock automated on 24 October 1988, Skerryvore in 1994, Muckle Flugga in 1995, the last Scottish keepers leaving on 31 March 1998 and the last Northern Lighthouse Board foghorn on 4 October 2005) in date order, with the gap in years between each one and the one before it, treating the years given alone as 1 January. Then find what share of the 212 years from the Board's 1786 act to the last keepers lies after the Bell Rock was automated, and how long each of the three towers kept its keepers.

View solution

EXERCISE 2

Use the sound-speed model to estimate distance from a flash-to-bang delay. Give the distance for delays of 2, 5, 10 and 20 s at 10 °C. Then find the percentage error in the distance if 343 m/s is used at 0 °C and at 30 °C, the time a signal takes to cover 3 nautical miles at 15 °C, and compare the model at 0 °C with the 332 m/s that the Paris Academy found in 1738 and Newton's 979 ft/s in metres per second.

View solution

EXERCISE 3

Work out the radar arithmetic. Find the echo delay in microseconds for targets at 0.5, 1, 6 and 12 nautical miles and the delay per nautical mile. Find the length in metres of a 25 microsecond racon reply and the range it spans on the display. Then for the two UK duty cycles (20 s on, 40 s off and 9 s on, 21 s off) find the cycle length, the cycles an hour, the silent seconds per hour and the longest time a ship could wait for a reply.

View solution

QUICK REFERENCE

- **Fog signals:** hand-struck bells and guns first; first purpose-made fog bell 1766 (Nidingen); steam foghorn at Partridge Island 1859 (Foulis); Trinity House siren at Ailsa Craig 1887; diaphone the early twentieth-century standard
- **Sound:** about 343 m/s in air at 20 °C, 331 m/s at 0 °C, 1,481 m/s in fresh water at 20 °C; model v = sqrt(1.4 R T / M)
- **Lightvessels:** first British one 1731 or 1734 (conflict); Pharos near Inchcape 1807; chain 6 ft per foot of water; bell heard at 15 miles in a test, 1 to 3 miles in practice

- **Racon:** replies on the radar's own frequency with a Morse letter; 25 microsecond reply; UK duty cycle about 30% (20 s on, 40 s off, or 9 s on, 21 s off)
- **Radar timing:** about 12.4 microseconds of echo delay per nautical mile, there and back
- **Automation:** Bell Rock 24 October 1988; Skerryvore 1994; Muckle Flugga 1995; Fair Isle South 31 March 1998, the last staffed Scottish light; last Board foghorn, Skerryvore, 4 October 2005
- **Conflicts:** first lightship 1731 or 1734; Trinity House lightvessels 8 (stated) or 11 (listed); Newton's error "about 15%" or 13.1% by my arithmetic

COMING NEXT

How Lighthouses Work 9 is the capstone, Plan a Light: choosing range, height and characteristic for a headland, a Python range calculator, reading a chart, and comparing three Scottish lights.

How Lighthouses Work

Chapter 9 · Plan a Light

This is the capstone, so for once we build instead of learn. Imagine you have been asked to put a light on a headland. Nobody has told you how tall to make the tower, how bright the lamp must be, or what rhythm it should flash, and each of those answers comes from a different part of How Lighthouses Work 1 to 8: the horizon and the inverse square law, the lamps and lenses, the flash characters, the rock and the weather. In this chapter we turn those pieces into one small Python range calculator, use it to meet a written brief, choose a characteristic that cannot be confused with the lights next door, read a chart label the way a mariner does, and finish by putting the three Stevenson rock lights side by side.

WHAT WAS RUN, AND WHAT TO TRUST

Every Python snippet in this chapter and in the three exercise solutions was run, and the output in the comments is what Python printed. The headland, the brief, the ship's eye height, the tower and ground heights, the 1 s flashes and the rule that two lights are confusable when their periods are within 3 s are *my inventions* and are labelled where they appear: the plan is a worked example, not a real design and not a navigation aid. The figures for the three real lights come from the infoboxes of the English Wikipedia pages for the Bell Rock Lighthouse, Skerryvore and Muckle Flugga Lighthouse, which I read directly for this chapter: focal heights of 28 m, 46 m and 66 m, listed ranges of 18, 23 and 22 nautical miles, and, for the Bell Rock only, the characteristic "Fl. W 5s 28m 18M" with a flash of 0.1 s and an eclipse of 4.9 s, a hyper-radial Fresnel lens and an intensity of 1,900,000 candela. The characteristics of Skerryvore and Muckle Flugga (Fl W 10s and Fl(2) W 20s), the automation years and the first-lit years are carried over from How Lighthouses Work 6 and 8 and were not looked up again. The chart-label convention comes from the Wikipedia page on light characteristics. All sums, ratios and conversions are my arithmetic. Where the sources disagree, the chapter says so.

The Brief

A planner starts with a statement of what the light is for. Mine is invented: a headland with a cliff top 20 m above the sea, a shipping lane offshore, and ships whose bridges are 8 m above the water. The light must be seen from 15 nautical miles in the standard weather of How Lighthouses Work 1 (meteorological visibility 10 nautical miles), and still from 6 nautical miles when the visibility falls to 2 nautical miles. Both figures are assumptions chosen to make the example work; a real brief would come from the authority and from the traffic.

Every decision in the brief answers to one of the earlier chapters, and the order matters, because the limits are independent: the horizon sets one, the brightness another, and what a sailor sees is the smaller of the two.

DECISION	THE QUESTION	THE TOOL	WHERE IT CAME FROM
Height of the light	How far does the horizon allow it to be seen?	geographic range, 2.084 (√H + √h) nautical miles	How Lighthouses Work 1
Intensity	How far does the brightness carry in haze?	Allard's law, solved by bisection	How Lighthouses Work 1 and 3
Optics	How do I turn a small flame or lamp into a bright beam?	a Fresnel lens of a chosen order	How Lighthouses Work 4
Characteristic	How will a sailor tell it from the next light?	Fl, Oc, Iso and group patterns	How Lighthouses Work 5
Site and structure	Can the tower be built, and who builds it?	cliff top or rock; tide-limited working	How Lighthouses Work 6 and 7
Fog	What happens when the light cannot be seen?	horn, bell, racon	How Lighthouses Work 8

A Range Calculator

The calculator is three functions. The first is the geographic range of How Lighthouses Work 1, with the light and the eye both contributing a horizon. The second finds the luminous range by bisection: it keeps halving an interval until the illuminance given by Allard's law falls to 0.2 microlux, and it takes the visibility as the distance at which 5% of the light survives, as that chapter did. The third returns the smaller of the two. The table it prints crosses three heights with two lamps, 100,000 cd and 1,000,000 cd (both assumed), for an 8 m eye.

```
# A range calculator for a proposed light: geographic range, luminous range and what is
import math

NM = 1852.0                # metres in a nautical mile
KM_PER_SQRT_M = 3.86       # horizon constant with standard refraction
THRESHOLD_LX = 2e-7        # 0.2 microlux, the nominal-range convention
```

```
def geographic_range_nm(light_m, eye_m):
    return KM_PER_SQRT_M * (math.sqrt(light_m) + math.sqrt(eye_m)) / (NM / 1000)

def luminous_range_nm(intensity_cd, visibility_nm):
    t = 0.05 ** (1 / visibility_nm)          # transmissivity per nautical mile
    lo, hi = 0.01, 200.0
    for _ in range(100):
        mid = (lo + hi) / 2
        lux = intensity_cd * t ** mid / (mid * NM) ** 2
        if lux > THRESHOLD_LX:
            lo = mid
        else:
            hi = mid
    return (lo + hi) / 2

def seen_range_nm(light_m, eye_m, intensity_cd, visibility_nm):
    return min(geographic_range_nm(light_m, eye_m), luminous_range_nm(intensity_cd, visi

EYE_M = 8                 # assumed bridge height of the ships the light is for
print("height | intensity | nominal | geographic | seen (10 nm) | seen (2 nm)")
for height in (20, 40, 60):
    for cd in (100_000, 1_000_000):
        nominal = luminous_range_nm(cd, 10)
        geo = geographic_range_nm(height, EYE_M)
        print(height, "m |", cd, "cd |", round(nominal, 1), "nm |", round(geo, 1), "nm |
              round(seen_range_nm(height, EYE_M, cd, 10), 1), "nm |", round(seen_range_n
# height | intensity | nominal | geographic | seen (10 nm) | seen (2 nm)
# 20 m | 100000 cd | 19.8 nm | 15.2 nm | 15.2 nm | 5.6 nm
# 20 m | 1000000 cd | 25.7 nm | 15.2 nm | 15.2 nm | 6.9 nm
# 40 m | 100000 cd | 19.8 nm | 19.1 nm | 19.1 nm | 5.6 nm
# 40 m | 1000000 cd | 25.7 nm | 19.1 nm | 19.1 nm | 6.9 nm
# 60 m | 100000 cd | 19.8 nm | 22.0 nm | 19.8 nm | 5.6 nm
# 60 m | 1000000 cd | 25.7 nm | 22.0 nm | 22.0 nm | 6.9 nm
```

Three things stand out. At 20 m a lamp ten times brighter changes nothing in clear weather: both lamps are cut off by the horizon at 15.2 nm. At 60 m the dimmer lamp is the limit (19.8 nm against a horizon of 22.0 nm), so there more intensity helps. And in poor visibility the height makes no difference at all: the 2 nm column reads 5.6 nm for the 100,000 cd lamp and 6.9 nm for the 1,000,000 cd lamp at every height, because the air, not the horizon, is the limit. Ten times

the intensity gave only about 1.2 times the range (6.9 divided by 5.6, from the rounded figures), which is the inverse square law at work again.

LIGHT	CLEAR WEATHER (10 NM)	POOR WEATHER (2 NM)
Low and dim (20 m, 100,000 cd)	15.2 nm, the horizon decides	5.6 nm, the brightness decides
Low and bright (20 m, 1,000,000 cd)	15.2 nm, the horizon decides	6.9 nm, the brightness decides
High and dim (60 m, 100,000 cd)	19.8 nm, the brightness decides	5.6 nm, the brightness decides
High and bright (60 m, 1,000,000 cd)	22.0 nm, the horizon decides	6.9 nm, the brightness decides

Meeting the Brief

Now run the calculator backwards. Given a required range, the focal height that just reaches it follows from solving the geographic formula for H, and the intensity that just reaches it can be found by searching on the luminous-range function itself; the search below halves the interval on a logarithmic scale, because the answer could be anywhere from a candle to millions of candela. The code does the clear-weather case and the poor-weather case separately, keeps the larger intensity, and then tests a tower of my choosing, 12 m on the 20 m cliff, against the horizon.

```
# Solve a brief: the lowest focal height and the lowest intensity that meet a required r
import math

NM = 1852.0
NM_PER_SQRT_M = 3.86 / (NM / 1000)              # about 2.084

def geographic_range_nm(light_m, eye_m):
    return NM_PER_SQRT_M * (math.sqrt(light_m) + math.sqrt(eye_m))

def luminous_range_nm(intensity_cd, visibility_nm):
    t = 0.05 ** (1 / visibility_nm)
    lo, hi = 0.01, 200.0
    for _ in range(100):
        mid = (lo + hi) / 2
        if intensity_cd * t ** mid / (mid * NM) ** 2 > 2e-7:
            lo = mid
        else:
```

```
            hi = mid
    return (lo + hi) / 2

def height_needed_m(range_nm, eye_m):
    return (range_nm / NM_PER_SQRT_M - math.sqrt(eye_m)) ** 2

def intensity_needed_cd(range_nm, visibility_nm):
    lo, hi = 1.0, 1e9
    for _ in range(200):
        mid = math.sqrt(lo * hi)                # halve the search on a log scale
        if luminous_range_nm(mid, visibility_nm) < range_nm:
            lo = mid
        else:
            hi = mid
    return hi

CLEAR_NM, POOR_NM, EYE_M = 15, 6, 8             # assumed brief: 15 nm in 10 nm visibili
print("focal height for a", CLEAR_NM, "nm horizon:", round(height_needed_m(CLEAR_NM, EYE_
cd_clear = intensity_needed_cd(CLEAR_NM, 10)
cd_poor = intensity_needed_cd(POOR_NM, 2)
print("intensity for", CLEAR_NM, "nm at 10 nm visibility:", round(cd_clear), "cd")
print("intensity for", POOR_NM, "nm at 2 nm visibility:", round(cd_poor), "cd")
need = max(cd_clear, cd_poor)
print("the brief needs at least", round(need), "cd; the clear-weather case decides:", cd_

GROUND_M, TOWER_M = 20, 12                      # assumed: cliff-top ground level and to
focal = GROUND_M + TOWER_M
geo = geographic_range_nm(focal, EYE_M)
print("focal height", focal, "m gives", round(geo, 1), "nm; margin over the brief:", rou
for factor in (1, 2, 5):
    cd = round(need * factor)
    print(factor, "x:", cd, "cd -> nominal", round(luminous_range_nm(cd, 10), 1), "nm; a
          round(luminous_range_nm(cd, 2), 1), "nm")
# focal height for a 15 nm horizon: 19.1 m
# intensity for 15 nm at 10 nm visibility: 13805 cd
# intensity for 6 nm at 2 nm visibility: 197562 cd
# the brief needs at least 197562 cd; the clear-weather case decides: False
# focal height 32 m gives 17.7 nm; margin over the brief: 2.7 nm
# 1 x: 197562 cd -> nominal 21.5 nm; at 2 nm visibility 6.0 nm
# 2 x: 395125 cd -> nominal 23.3 nm; at 2 nm visibility 6.4 nm
# 5 x: 987812 cd -> nominal 25.7 nm; at 2 nm visibility 6.9 nm
```

The horizon needs a focal height of only 19.1 m for 15 nm, and the 15 nm brightness needs only 13,805 cd. The poor-weather requirement is the hard one: 6 nm in 2 nm visibility needs 197,562 cd, about fourteen times as much (by my arithmetic from the printed figures). The line "the clear-weather case decides: False" says exactly that, and it is the lesson of the whole capstone: a lamp is sized for the bad night, a tower is sized for the good one. The 32 m focal height (20 m of cliff plus 12 m of tower) gives 17.7 nm, a margin of 2.7 nm. Doubling the minimum intensity to 395,125 cd lifts the nominal range to 23.3 nm but the poor-weather range only from 6.0 to 6.4 nm, and five times the minimum reaches 6.9 nm, so the extra lamp buys very little where it is needed. I take a round 400,000 cd, about twice the minimum and roughly a fifth of the Bell Rock's 1,900,000 cd (my arithmetic). A lens of the kind in How Lighthouses Work 4 would be what turns a lamp into that candlepower; the plan does not choose an order, and the sizes of lens in that chapter's model were illustrations, not measurements.

Choosing a Characteristic

A sailor identifies a light by its rhythm, so a new light must not look like its neighbours. For the example I place the headland between three lights with the characteristics of the Bell Rock, Skerryvore and Muckle Flugga (Fl W 5s, Fl W 10s and Fl(2) W 20s). This is an invented neighbourhood: the real lights are nowhere near each other. The code tests six candidate characteristics against two things: the rules for flashing lights quoted in How Lighthouses Work 5 (a single flash has a period of 2 to 15 s with darkness at least three times the flash; a group of two has a period of at most 20 s and a group of three or more at most 30 s), and a confusion test that is entirely my own rule: two lights are confusable if they have the same class and the same number of flashes and their periods are within 3 s of each other. The source gives no such number; it is a simple way to make the idea concrete.

```
# Pick a characteristic that no neighbour already uses, and check it against the flash r
NEIGHBOURS = {"Fl W 5s": ("Fl", 1, 5), "Fl W 10s": ("Fl", 1, 10), "Fl(2) W 20s": ("Fl",
CANDIDATES = {"Fl W 5s": ("Fl", 1, 5), "Fl W 12s": ("Fl", 1, 12), "Fl(2) W 10s": ("Fl",
              "Fl(3) W 15s": ("Fl", 3, 15), "Fl(2) W 20s": ("Fl", 2, 20), "Iso W 4s": ("
FLASH_S, GAP_S = 1, 1          # assumed: 1 s flashes with 1 s gaps inside a group

def passes_rules(cls, n, period):
    if cls == "Iso":
        return True
    if n == 1:
        return 2 <= period <= 15 and period - FLASH_S >= 3 * FLASH_S    # single flash:
```

```
    limit = 20 if n == 2 else 30
    return period <= limit and period > n * FLASH_S + (n - 1) * GAP_S   # group: period

def confusable(a, b):
    return a[0] == b[0] and a[1] == b[1] and abs(a[2] - b[2]) < 3       # same class and

def light_share(cls, n, period):
    return 0.5 if cls == "Iso" else n * FLASH_S / period

for name, c in CANDIDATES.items():
    clashes = [nb for nb, v in NEIGHBOURS.items() if confusable(c, v)]
    verdict = "ok" if passes_rules(*c) and not clashes else "reject"
    print(name, "| rules:", passes_rules(*c), "| clashes with:", clashes or "none",
          "| lit", round(100 * light_share(*c)), "% ->", verdict)
# Fl W 5s | rules: True | clashes with: ['Fl W 5s'] | lit 20 % -> reject
# Fl W 12s | rules: True | clashes with: ['Fl W 10s'] | lit 8 % -> reject
# Fl(2) W 10s | rules: True | clashes with: none | lit 20 % -> ok
# Fl(3) W 15s | rules: True | clashes with: none | lit 20 % -> ok
# Fl(2) W 20s | rules: True | clashes with: ['Fl(2) W 20s'] | lit 10 % -> reject
# Iso W 4s | rules: True | clashes with: none | lit 50 % -> ok
```

Fl W 5s is the Bell Rock's own character and is rejected; Fl W 12s is within 3 s of Fl W 10s; Fl(2) W 20s is identical to Muckle Flugga's. That leaves Fl(2) W 10s, Fl(3) W 15s and Iso W 4s. All three would do; I choose Fl(3) W 15s, three flashes in 15 s, because a group of three is easy for a tired watchkeeper to count and no neighbour shows one. It is lit 20% of the time (three 1 s flashes in 15 s), the same share as the Fl(2) W 10s option, and its group of three needs a period of at most 30 s, which it meets easily.

Reading a Chart

The finished design appears on a chart as one line of text. The page on light characteristics gives the convention: the class and group, the colour, the period, the height of the light above the chart datum (usually based on high water), and a range in nautical miles written with an M. Its worked example, "Gp Oc(3) W 10s 15m 10M", is described as a group occulting white light with a period of 10 s, 15 m high, whose nominal range is 10 nautical miles. The page is not consistent about the range: in the list it calls the range "the range in which the light is visible", and in the example it calls it the nominal range. As How Lighthouses Work 1 explained, the nominal range

ignores the horizon, so I follow the worked example. Our design therefore reads "Fl(3) W 15s 32m 23M": 32 m is the focal height, and 23 is the 23.3 nm of the 395,125 cd run, rounded.

The code below parses a label with a regular expression, works out how far a 5 m eye can see the light, and says whether the horizon or the brightness is the limit. It then does the reverse: given a list of the times at which flashes began, it finds the long gap in the pattern and counts the flashes between two long gaps.

```
# Read a chart label, then identify a light from stopwatch readings.
import math
import re

NM_PER_SQRT_M = 2.084

def parse(label):
    pattern = r"(?P<cls>Fl|LFl|Iso|Oc)(?:\((?P<n>\d+)\))?\s+(?P<col>[WRG])\s+(?P<period>
    d = re.match(pattern, label).groupdict()
    return d["cls"], int(d["n"] or 1), d["col"], int(d["period"]), int(d["h"]), int(d["r

for label in ("Fl W 5s 28m 18M", "Fl W 10s 46m 23M", "Fl(2) W 20s 66m 22M", "Fl(3) W 15s
    cls, n, col, period, h, rng = parse(label)
    geo = NM_PER_SQRT_M * (math.sqrt(h) + math.sqrt(5))
    limit = "horizon" if geo < rng else "brightness"
    print(label, "| flashes", n, "| period", period, "s | 5 m eye sees to", round(geo, 1

def identify(starts):
    gaps = [round(b - a, 1) for a, b in zip(starts, starts[1:])]
    long_gap = max(gaps)
    first = gaps.index(long_gap)
    second = gaps.index(long_gap, first + 1)
    n = second - first                                  # flashes between two long gaps
    period = round(sum(gaps[first:second]), 1)
    return n, period

watch = [3.0, 5.0, 7.0, 18.0, 20.0, 22.0, 33.0, 35.0, 37.0, 48.0]    # flash start times
print("gaps:", [round(b - a, 1) for a, b in zip(watch, watch[1:])])
print("flashes per group and period:", identify(watch))
# Fl W 5s 28m 18M | flashes 1 | period 5 s | 5 m eye sees to 15.7 nm | limited by horizo
# Fl W 10s 46m 23M | flashes 1 | period 10 s | 5 m eye sees to 18.8 nm | limited by hori
# Fl(2) W 20s 66m 22M | flashes 2 | period 20 s | 5 m eye sees to 21.6 nm | limited by h
# Fl(3) W 15s 32m 23M | flashes 3 | period 15 s | 5 m eye sees to 16.4 nm | limited by h
# gaps: [2.0, 2.0, 11.0, 2.0, 2.0, 11.0, 2.0, 2.0, 11.0]
```

```
# flashes per group and period: (3, 15.0)
```

For a 5 m eye the first three lights are limited by the horizon, at 15.7, 18.8 and 21.6 nm, against printed ranges of 18, 23 and 22 nm, which is the finding of How Lighthouses Work 6 again. Our own design is also limited by the horizon (16.4 nm for a 5 m eye, against 23 nm printed), which is a reminder that a bigger number on the chart is not a longer view. The stopwatch readings contain gaps of 2 s, 2 s and 11 s repeated, so the pattern is three flashes with a period of 2 + 2 + 11 = 15 s, which is the character we chose. This simple version looks only at when flashes begin, so it cannot tell an isophase light from an occulting one, or a short flash from a long one; a mariner would also note how long each flash lasts.

Three Scottish Lights Compared

Finally, put the planning tool to work on real lights. The data are the focal heights and listed ranges read from the pages, the characteristics from How Lighthouses Work 6, and the years from How Lighthouses Work 6 and 8. For each light the code finds the horizon for a 5 m eye, compares it with the listed range, works out how high an eye would have to be for the listed range to be the horizon, and counts the flashes an hour.

```
# Compare three Scottish rock lights, using the figures from the earlier chapters.
import math

NM_PER_SQRT_M = 2.084
LIGHTS = (   # (name, focal height m, listed range nm, characteristic, period s, flashes
    ("Bell Rock", 28, 18, "Fl W 5s", 5, 1, 1811, 1988),
    ("Skerryvore", 46, 23, "Fl W 10s", 10, 1, 1844, 1994),
    ("Muckle Flugga", 66, 22, "Fl(2) W 20s", 20, 2, 1858, 1995),
)
print("name | geographic (5 m eye) | listed | listed/geographic | eye needed | flashes p
for name, h, rng, ch, period, n, lit, auto in LIGHTS:
    geo = NM_PER_SQRT_M * (math.sqrt(h) + math.sqrt(5))
    eye = (rng / NM_PER_SQRT_M - math.sqrt(h)) ** 2
    print(name, "|", round(geo, 1), "nm |", rng, "nm |", round(rng / geo, 2), "|", round
          round(3600 / period * n), "|", auto - lit)
tall = max(LIGHTS, key=lambda x: x[1])
low = min(LIGHTS, key=lambda x: x[1])
```

```
print("height ratio", tall[0], "to", low[0], ":", round(tall[1] / low[1], 2))
print("geographic range ratio, 5 m eye:", round((math.sqrt(tall[1]) + math.sqrt(5)) / (m
print("Bell Rock flash of 0.1 s in 5 s: lit", round(100 * 0.1 / 5, 1), "% of the time")
print("a 0.1 s flash at 5 s per turn matches a beam", round(0.1 / 5 * 360, 1), "degrees
print("model hyper-radial beam of 0.86 degrees at 5 s per turn lasts", round(0.86 / 360
# name | geographic (5 m eye) | listed | listed/geographic | eye needed | flashes per ho
# Bell Rock | 15.7 nm | 18 nm | 1.15 | 11.2 m | 720 | 177
# Skerryvore | 18.8 nm | 23 nm | 1.22 | 18.1 m | 360 | 150
# Muckle Flugga | 21.6 nm | 22 nm | 1.02 | 5.9 m | 360 | 137
# height ratio Muckle Flugga to Bell Rock : 2.36
# geographic range ratio, 5 m eye: 1.38
# Bell Rock flash of 0.1 s in 5 s: lit 2.0 % of the time
# a 0.1 s flash at 5 s per turn matches a beam 7.2 degrees wide
# model hyper-radial beam of 0.86 degrees at 5 s per turn lasts 0.012 s
```

	BELL ROCK	SKERRYVORE	MUCKLE FLUGGA
Focal height	28 m	46 m	66 m
Characteristic	Fl W 5s	Fl W 10s	Fl(2) W 20s
Listed range	18 nm	23 nm	22 nm
Horizon, 5 m eye	15.7 nm	18.8 nm	21.6 nm
Listed range / horizon	1.15	1.22	1.02
Flashes an hour	720	360	360
First lit to automation	177 years	150 years	137 years

Muckle Flugga is the only one whose listed range matches the geometry (a ratio of 1.02, an eye of 5.9 m), which suggests that its range is a horizon figure for a low boat, while the Bell Rock and Skerryvore ranges are larger than any such horizon and would have to be nominal ranges or assume a high eye. I cannot tell which from my sources, as before. The tallest light, Muckle Flugga, is 2.36 times the height of the Bell Rock but sees only 1.38 times as far for a 5 m eye, the square root law at work. The Bell Rock's own infobox gives a flash of 0.1 s in a 5 s period, so the light is on for 2.0% of the time. If the lens turns once in 5 s, which I am assuming, that flash matches a beam 7.2 degrees wide, whereas the model hyper-radial beam of How Lighthouses Work 4 (0.86 degrees) would last only 0.012 s. The page does not give the beam width or the rotation period, so the tension noted in How Lighthouses Work 5 between narrow beams and

second-long flashes remains unresolved. Most of the other numbers in this chapter are tidier than that one.

Common mistakes

Sizing the lamp for good weather when the bad night is the harder requirement; buying a taller tower to fix a poor-visibility problem, or a brighter lamp to fix a horizon problem; quoting the chart range as the distance at which a small boat first sees the light; forgetting that the chart height is usually from high water, so the light stands higher at low tide; choosing a characteristic without checking the neighbours; reading Fl(3) as three seconds instead of three flashes; mixing up the number of flashes in a group with the period; calling any light that flashes a fixed light; treating my 3 s confusion rule, the 1 s flashes and the invented headland as sourced facts; and treating a chapter of examples as a design for a real light.

EXERCISE 1

Plan a light for a second headland. The ground is 30 m above the sea, the ships' eyes are 10 m up, the light must be seen at 20 nm in standard weather (10 nm visibility) and at 8 nm in 2 nm visibility. Find the focal height needed for the horizon and the tower that gives it, the horizon range of towers of 10, 15, 20 and 25 m, the intensity each weather case needs and which one decides, and the nominal range for 1, 2, 4 and 10 times the larger intensity.

View solution

EXERCISE 2

Choose a characteristic between neighbours that show Fl(2) W 10s, Fl W 8s and Oc W 6s. Test the six candidates Fl(3) W 15s, Fl W 10s, Fl(2) W 12s, Oc W 6s, Fl W 3s and Iso W 8s against the single-flash and group rules and the 3 s confusion rule. Then write a function that identifies a light from the start times of its flashes, and use it on three stopwatch lists, and give the shortest watch that shows two full periods of the slowest candidate.

View solution

EXERCISE 3

Read five chart labels: Fl W 5s 28m 18M, Oc W 6s 20m 12M, Iso W 4s 15m 10M, Fl(2) W 10s 45m 25M and LFl W 12s 30m 16M. Parse each, and for eyes at 2, 5 and 10 m give the horizon range and say whether the horizon or the brightness limits it. Find how many minutes a boat at 6 knots takes to reach

each light from first sight with a 5 m eye, and how much farther each light can be seen at low water if the tide is 4 m below the chart datum (an assumption).

View solution

QUICK REFERENCE

- **The plan:** choose the horizon (height), the brightness (intensity), the characteristic and the site, in that order; what is seen is the smaller of the horizon and luminous ranges
- **Formulas:** geographic range = 2.084 (√H + √h) nm with heights in metres; luminous range by Allard's law to 0.2 microlux, solved by bisection
- **Worked example:** 15 nm clear and 6 nm in 2 nm visibility needs 19.1 m of focal height and 197,562 cd; the poor-weather case decides the lamp
- **Characteristic:** Fl(3) W 15s chosen against neighbours Fl W 5s, Fl W 10s and Fl(2) W 20s; the 3 s confusion rule is my own
- **Chart label:** class, colour, period, height above chart datum (usually high water), range in M; written "Fl(3) W 15s 32m 23M"
- **Real lights:** Bell Rock Fl W 5s 28m 18M, 1,900,000 cd, flash 0.1 s; Skerryvore 46 m, 23 nm; Muckle Flugga 66 m, 22 nm
- **Conflicts:** the light characteristics page calls the M range both "visible" and "nominal"; listed ranges for the Bell Rock and Skerryvore exceed the horizon for a 5 m eye; narrow beams against flashes of 0.1 s or more (unresolved)

COMING NEXT

This is the last chapter in the plan for How Lighthouses Work, so there is no further chapter to preview. The course is complete: from the inverse square law and the horizon to the keepers and the automated lights, you now have the tools to read a lighthouse, and to plan one.