

COURSE

How Bridges Work

Forces, beams, arches, trusses, cantilevers, cables and the three Forth bridges

10 Chapters

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HOW BRIDGES WORK

Contents

Chapter 1	Forces and Loads	2
Chapter 2	Beams and Simple Spans	9
Chapter 3	Arches and Early Bridges	17
Chapter 4	Trusses	24
Chapter 5	Materials	32
Chapter 6	Cantilevers and the Forth Bridge	40
Chapter 7	Suspension Bridges	48
Chapter 8	Cable-Stayed Bridges	56
Chapter 9	Wind, Water and Failure	64
Chapter 10	Capstone: Design and Test a Bridge	73

How Bridges Work 🏆

Chapter 1 · Forces and Loads

Within sight of each other, on the Firth of Forth, stand three bridges that carry traffic across the same water in three completely different ways: a steel rail bridge from 1890 built as a series of balanced cantilevers, a suspension road bridge from 1964, and a cable-stayed crossing from 2017. All three do the same job. They take a load that is out in the middle of the air and pass it, step by step, to the ground on either side. This course follows that idea from the simplest beam to the Forth bridges themselves, and this first chapter gives you the vocabulary: the five kinds of force, the two kinds of load, and the picture of a **load path** that every later chapter builds on.

> **WHAT WAS RUN**
>
> Every Python snippet in this chapter and in the three exercise solutions was executed, and the output shown in the comments is what Python printed. You only need Python 3 and its standard library.

The Job of a Bridge

A bridge has one rule: **whatever weight lands on it must end up in the ground**. Nothing else matters if that fails. Everything in this course, from the shape of an arch to the paint on the Forth Bridge, is about making that journey safe and economical. Engineers describe the journey with two ideas: *loads* (what pushes on the bridge) and *forces* (what happens inside the structure as it resists them).

Two Kinds of Load

A **load** is anything that pushes on the structure. They are sorted by how they behave:

LOAD	WHAT IT IS	EXAMPLE	BEHAVIOUR
Dead load	the weight of the bridge itself	steel, concrete deck, road surface, railings	always there, never changes
Live load	the weight of things that use the bridge	cars, lorries, trains, pedestrians	moves and varies

LOAD	WHAT IT IS	EXAMPLE	BEHAVIOUR
Environmental load	weather and nature	wind, temperature change, snow, water flow, earthquakes	varies, sometimes suddenly

The surprise for beginners is how big the dead load is. For a long bridge, carrying its own weight is the hardest job: a longer span has to support more steel, and that extra steel has to be supported too. Much of bridge history, including the cantilever design of the Forth Bridge, is a story of using material more cleverly so the bridge wastes less strength carrying itself.

Mass Is Not Force

Before any calculation, one trap. Mass (kilograms) is how much matter there is. Force (newtons, N, or kilonewtons, kN) is how hard that mass pulls down under gravity. To convert, multiply by the gravitational acceleration, about 9.81 m/s^2: a 1000 kg mass weighs about 9.81 kN. Engineers also speak loosely about "tonnes" of load, but the calculation must be in force units, and mixing the two is the commonest error in a first structures course.

```
G = 9.81                        # m/s2, turns mass into weight

def kn(kg):
    return kg * G / 1000        # kg -> kilonewtons

deck_area = 12 * 8              # a 12 m x 8 m deck, in m2
slab_mass = deck_area * 0.25 * 2400   # 0.25 m of concrete at 2400 kg/m3
dead = kn(slab_mass)
live = 5 * deck_area                # assume 5 kN on every m2
print(round(slab_mass), round(dead, 1), live, round(dead + live, 1), round((dead + live)
# 57600 565.1 480 1045.1 261.3
```

Read the output as: the slab has a mass of 57,600 kg, which is a dead load of 565.1 kN; the assumed live load adds 480 kN, giving 1045.1 kN in total; shared between four supports that is 261.3 kN each. (The live load of 5 kN/m^2 is a round figure chosen for this example. Real design codes set the values for each bridge type.)

Five Kinds of Force

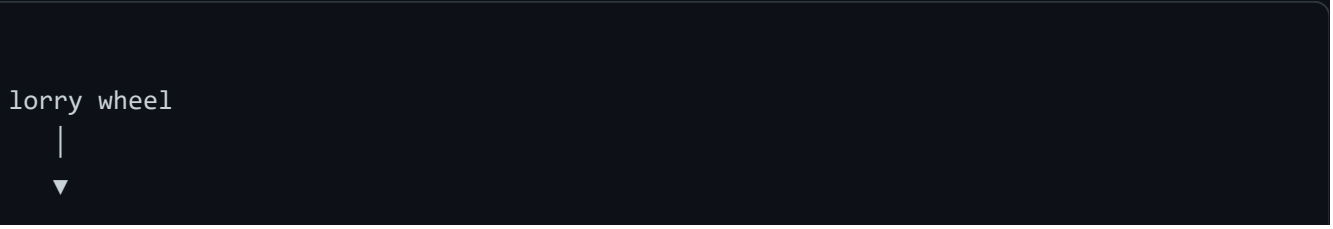
Loads come from outside. **Forces** are what they cause *inside* each member of the structure. There are five basic kinds, and every bridge in this course is an arrangement of them:

FORCE	WHAT IT DOES	EVERYDAY PICTURE	WHERE IN A BRIDGE
Tension	pulls the material apart	a tug of war rope	suspension cables, stay cables, hangers, bottom chord of a truss
Compression	squeezes the material together	standing on a can	stone arches, towers, piers, top chord of a truss
Shear	slides one part past the next	scissors cutting paper	near supports; bolts and rivets joining members
Bending	curves the member: top squeezed, bottom stretched	a plank across a ditch	every beam and deck
Torsion	twists the member	wringing out a cloth	curved decks, off-centre loads, boxes in wind

Bending is really tension and compression working together: when a plank sags, its lower face is stretched and its upper face is squeezed. That is why a deep beam is stronger than a shallow one of the same weight (How Bridges Work 2). Materials differ in which force they prefer. Stone and concrete handle compression well and tension badly; steel handles both; a rope handles only tension. Much of bridge design is choosing the right material for the right force.

The Load Path

Follow a lorry as it crosses a bridge. Its wheels press on the road surface. The surface passes the weight to the deck, the deck to the beams or girders under it, the beams to the supports (piers, towers, abutments), and the supports to the foundations, and finally into the rock or soil. That chain is the **load path**, and a bridge fails if any link is too weak or missing.



```
road surface
   |
   ▼
deck slab
   |
   ▼
beams / girders          ← bending and shear
   |
   ▼
piers or towers          ← compression
   |
   ▼
foundations
   |
   ▼
rock or soil
```

Each type of bridge is a different answer to the question "which path should the load take?". A beam sends it sideways along the member and down through its ends. An arch turns the load into compression along a curve (How Bridges Work 3). A suspension bridge hangs the deck from cables in tension, which run to towers and anchorages (How Bridges Work 7). A cable-stayed bridge runs straight lines of tension from deck to tower (How Bridges Work 8). Learn to ask "where does this weight go next?" and any bridge becomes legible.

Balance: Reactions at the Supports

A structure that is standing still has forces that balance: what pushes down must be matched by what pushes up. At the two ends of a simple beam the ground pushes up with **reactions**, and they must add up to the load. Where the load sits decides how the reaction is shared: the closer the load is to a support, the more of it that support carries. For a single point load on a beam of span L, with the load a distance `a` from support A, taking moments about A gives the reaction at B, and the reaction at A is what is left:

```
def reactions(span, load, a):
    rb = load * a / span          # moments about A
    ra = load - rb                # the rest goes to A
    return ra, rb
```

```
print(reactions(10, 60, 3))        # (42.0, 18.0)  load near A: A carries more
print(reactions(10, 60, 5))        # (30.0, 30.0)  load at midspan: shared equally
```

A 60 kN load three metres from A on a 10 m beam gives 42 kN to A and 18 kN to B, which add up to 60 kN. At midspan the load splits evenly. These two numbers are the first thing an engineer finds when checking any beam, because everything else (bending, shear, support design) follows from them.

Stress: How Hard the Material Works

A force alone does not tell you whether a member will fail: a 100 kN pull is nothing to a thick steel bar and fatal to a thin one. What matters is the force *per unit of cross-section*, called **stress**: stress = force ÷ area. With force in newtons and area in square millimetres, the answer comes out directly in megapascals (MPa), the unit in which material strengths are quoted. Ordinary structural steel yields (starts to deform permanently) at roughly 250 to 350 MPa, depending on the grade. In design you never aim for the limit: you divide the strength by a **factor of safety** (say 2) so that the real working stress is only half of what the material can take.

```
import math

def stress_mpa(force_kn, area_mm2):
    return force_kn * 1000 / area_mm2     # N per mm2 = MPa

area = math.pi * (20 / 2) ** 2              # a 20 mm diameter rod
s = stress_mpa(120, area)
print(round(area, 1), round(s, 1), round(250 / s, 2))
# 314.2 382.0 0.65

need = 120000 * 2 / 250                   # area for a factor of safety of 2
d = 2 * math.sqrt(need / math.pi)
print(round(need), round(d, 2))
# 960 34.96
```

The 20 mm rod would be stressed to 382 MPa under a 120 kN pull, well beyond a steel of 250 MPa (the "factor" of 0.65 means it would fail). To reach a safety factor of 2 the rod needs an area of 960 mm^2, which is a diameter of about 35 mm. Doubling the diameter would multiply the area by four, so the cost of strength grows quickly, and this is why engineers care so much about shape and not just size.

> Common mistakes
> Mixing mass and force (kg against kN); forgetting that the dead load is always there, so the live load is an addition and not the whole story; and using the diameter instead of the area in a stress calculation (stress depends on the area, which grows with the *square* of the diameter).

Where This Leads

You now have the toolkit: five forces, three kinds of load, a load path from wheel to rock, reactions that balance the load, and stress as force over area. The next chapters put these to work on one structure type at a time, beginning with the simplest of all: a beam across a gap. The three Forth bridges will return in the later chapters as worked examples of the cantilever, the suspension bridge and the cable-stayed bridge.

EXERCISE 1

A footbridge deck is 20 m long and 3 m wide, made of a 0.2 m thick concrete slab (2400 kg/m^3), and is designed for a live load of 4 kN on every square metre. It rests on two supports, one at each end. Write a program that works out the mass of the slab, the dead load and the live load in kN, the total, and how much each support carries. Which is bigger, the dead load or the live load, and what does that suggest about the design?

View solution

EXERCISE 2

A beam of span 12 m carries two point loads: 30 kN at 4 m from the left support A and 50 kN at 9 m from A. Write `reactions(span, loads)` that takes a list of (force, distance) pairs and returns the reactions at A and B by taking moments about A. Check your answer by confirming that the reactions add up to the total load, and by taking moments about B instead. Predict first which support carries more.

View solution

EXERCISE 3

A steel hanger carries a steady tension of 200 kN. The steel yields at 355 MPa and the design needs a factor of safety of 1.5. Work out the minimum cross-sectional area, then test stock rods of diameter 25, 30, 32, 36 and 40 mm: print the area, the stress and the factor of safety for each, and report the smallest stock rod that is safe.

View solution

QUICK REFERENCE

- **Loads:** dead (the bridge itself), live (traffic), environmental (wind, temperature, water)
- **Forces:** tension, compression, shear, bending, torsion
- **Load path:** wheel → surface → deck → beams → piers or towers → foundations → ground
- **Reactions:** they add up to the total load; the nearer support carries more
- **Stress:** force ÷ area; N/mm^2 = MPa
- **Factor of safety:** strength ÷ working stress
- **Units:** kg is mass; kN is force; multiply kg by 9.81 and divide by 1000

COMING NEXT

How Bridges Work 2 takes the simplest structure of all, a beam across a gap: why bending makes depth matter more than width, why the I-beam has its shape, and how the span limits what a plain beam can do.

How Bridges Work

Chapter 2 · Beams and Simple Spans

The oldest bridge is a log laid across a stream, and the idea has not changed: a straight member rests on a support at each end and carries the load between them. How Bridges Work 1 found the *reactions* at the two ends. This chapter asks what happens *inside* the beam between them: how bending and shear build up along its length, why a deep beam beats a shallow one, why an I-beam has the shape it does, and why a plain beam runs out of strength long before it reaches the spans of the Forth bridges.

> **WHAT WAS RUN**
>
> Every Python snippet in this chapter and in the three exercise solutions was executed, and the output shown in the comments is what Python printed. You only need Python 3 and its standard library.

Simply Supported

A **simply supported beam** rests on a support at each end. The supports push up but do not grip the beam, so the ends are free to tilt as the beam sags. The distance between the supports is the **span**. This is the simplest case to analyse, and the one that every bridge-building student meets first. Two quantities describe what the load does to it along its length:

QUANTITY	SYMBOL	WHAT IT IS	LARGEST WHERE
Shear force	V	the sliding force across a cut through the beam, equal to the net upward force on one side of the cut	at the supports
Bending moment	M	the turning effect at a cut, equal to the sum of force × distance for everything on one side	at midspan, for a symmetric load

For the two cases you will meet most often, the maximum values come from standard formulas, where `w` is a uniform load in kN per metre and `P` a single load at midspan in kN, on a span `L` in metres:

LOAD	MAX SHEAR (AT THE SUPPORTS)	MAX MOMENT (AT MIDSPAN)
Uniform load `w` over the whole span	$wL / 2$	$wL^2 / 8$

LOAD	MAX SHEAR (AT THE SUPPORTS)	MAX MOMENT (AT MIDSPAN)
Point load `P` at midspan	P / 2	PL / 4

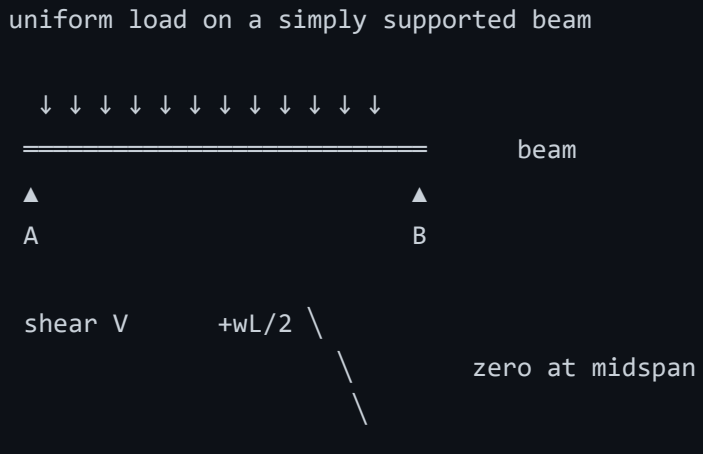
```
def udl_beam(w, span):                    # w in kN/m, span in m
    return w * span**2 / 8, w * span / 2  # (max moment kN m, max shear kN)

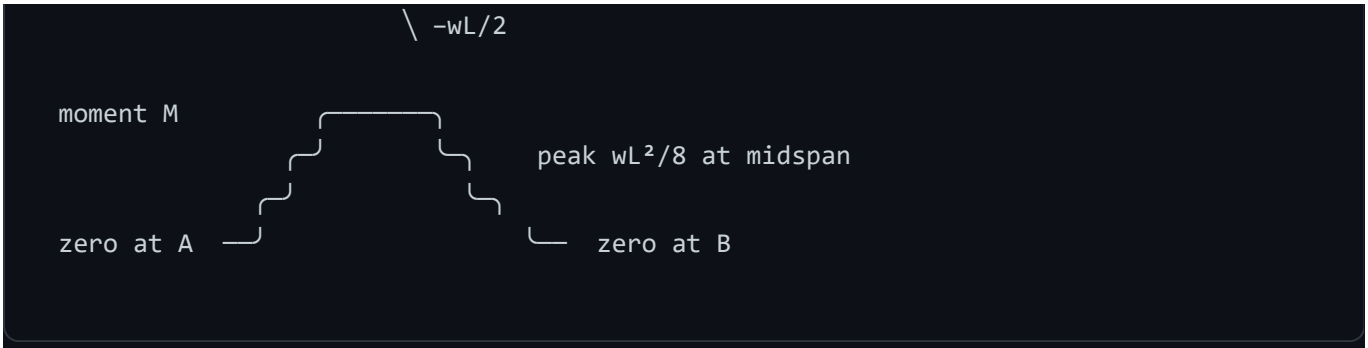
def point_beam(p, span):                  # one load p (kN) at midspan
    return p * span / 4, p / 2

print(udl_beam(20, 10))      # (250.0, 100.0)
print(point_beam(200, 10))   # (500.0, 100.0)
print(udl_beam(20, 20))      # (1000.0, 200.0)
```

Two lessons are in those three lines. First, **how a load is spread matters**: 20 kN/m over 10 m is a total of 200 kN, and it produces a moment of 250 kN·m, while the same 200 kN concentrated at midspan produces twice that (500 kN·m). Spreading a load out is kind to a beam. Second, **span is expensive**: with the same load per metre, doubling the span from 10 m to 20 m multiplies the moment by four (250 to 1000 kN·m), because the moment grows with the *square* of the span.

The shape of the two diagrams is worth picturing. For a uniform load, shear starts at its maximum at one support, falls in a straight line through zero at midspan, and reaches its maximum again, with the opposite sign, at the other support. The moment is zero at both ends (the beam is free to tilt there, so nothing resists turning) and builds in a smooth curve to its peak at midspan.





Bending Stress: Why Depth Matters

How Bridges Work 1 said that bending is tension and compression working together. Here is how. As a beam sags, the top face is squeezed and the bottom face is stretched. Somewhere between them is a layer that is neither: the **neutral axis**. The stress in the material is zero there and grows in proportion to the distance from it, reaching its largest at the top and bottom faces. The formula that links everything is:

> **THE BENDING FORMULA**
>
> **stress = M × y / I**, where `M` is the bending moment, `y` is the distance from the neutral axis to the face you are looking at, and `I` is the **second moment of area** of the cross-section, a number that measures how well the shape resists bending. For a rectangle of width `b` and depth `h`, `I = b×h³/12`.

Look at the cube in that formula. Depth appears as `h³`, width only as `b`. The same amount of material arranged deeper is far more resistant to bending:

```
def rect_i(b, h):
    return b * h**3 / 12                # mm^4

def bend_stress(m_knm, i_mm4, y_mm):
    return m_knm * 1e6 * y_mm / i_mm4   # N/mm2 = MPa

M = 10                                  # kN m
for b, h in [(100, 100), (50, 200), (200, 50)]:
    i = rect_i(b, h)
    print(b, h, b * h, round(i), round(bend_stress(M, i, h / 2), 1))
# 100 100 10000 8333333 60.0
# 50 200 10000 33333333 30.0
```

```
# 200 50 10000 2083333 120.0
```

All three bars have the same cross-sectional area (10,000 mm^2), so they use the same amount of steel and weigh the same. Yet under the same 10 kN·m moment the stress is 60 MPa in the square bar, 30 MPa when the bar is turned on edge (50 wide, 200 deep), and 120 MPa when it is laid flat. **Same material, four times the stress between best and worst orientation.** This is why floor joists stand on edge and why a plank across a ditch bends less when you turn it on its side.

The I-Beam: Material Where It Works

Look again at the stress picture. The material near the neutral axis is hardly stressed: all the heavy work is at the top and bottom faces. A solid rectangle therefore wastes steel in the middle. The **I-beam** (and its relatives, the plate girder and the box girder) solves this by concentrating the material in two **flanges** far from the neutral axis, joined by a thin **web** that keeps them apart and carries the shear.

```
def i_section(bf, tf, tw, depth):
    inner = depth - 2 * tf
    return (bf * depth**3 - (bf - tw) * inner**3) / 12

def area_i(bf, tf, tw, depth):
    return 2 * bf * tf + tw * (depth - 2 * tf)

a = area_i(150, 12, 8, 300)              # flanges 150 x 12, web 8, depth 300
i = i_section(150, 12, 8, 300)
side = a ** 0.5                          # a square bar of the same area
i_sq = side**4 / 12
print(a, round(i))                       # 5808 88709184
print(round(side, 1), round(i_sq), round(i / i_sq, 1))
# 76.2 2811072 31.6
```

This I-section has a cross-section of 5,808 mm^2. A solid square bar with the same area is 76.2 mm on a side. The I-section's `I` is 31.6 times larger, which means it resists bending over thirty times better for exactly the same weight of steel. That is the whole reason for the shape.

The idea is old. When Robert Stephenson built the Britannia Bridge over the Menai Strait in the 1840s, the trains ran through enormous rectangular tubes of wrought-iron plate: box girders, which are hollow beams with the same logic as the I-section. The same thought appears in the box-shaped decks of many modern bridges. (This history is from general knowledge, not from a source checked for this chapter.)

> What the formula leaves out
> A tall thin web can buckle sideways, and a deep beam can twist and buckle away from its loaded plane unless it is braced. Real designs check these too. The formula here gives you the first and most important answer, the bending stress, and nothing more.

Deflection: How Much It Sags

A beam can be strong enough and still be unusable if it sags too much: a deck that dips visibly is uncomfortable to cross and cracks its surfacing. The middle of a simply supported beam under a uniform load sags by

> **DEFLECTION OF A UNIFORMLY LOADED SIMPLE BEAM**
> **$\delta = 5\,w\,L^4 / (384\,E\,I)$**, where `E` is the material's stiffness (Young's modulus; about 200,000 MPa for steel) and `I` is the same second moment of area as before. Deflection grows with the *fourth* power of the span.

```
E = 200e3                                  # MPa, steel

def deflection_mm(w, span_m, i_mm4):       # w in kN/m (the same as N/mm)
    L = span_m * 1000
    return 5 * w * L**4 / (384 * E * i_mm4)

for span in (4, 8, 16):
    d = deflection_mm(10, span, 8.0e7)
    print(span, round(d, 1), round(span * 1000 / d))
# 4 2.1 1920
# 8 33.3 240
# 16 533.3 30
```

The three columns are the span in metres, the sag in millimetres, and the span divided by the sag. A 4 m span sags 2.1 mm (about span/1920): invisible. At 8 m it sags 33.3 mm (span/240), and at 16 m it sags 533 mm (span/30), more than half a metre. Each doubling of span multiplies the sag by sixteen. A common design target for live-load sag is around span/360, which the 8 m case already misses. Stiffness, not strength, often decides the size of a beam.

The Span Limit of a Plain Beam

There is a harder limit still. A beam must carry its own weight, and that weight grows with the length. For the same section, the self-weight moment `wL²/8` grows with the square of the span, so there is a span at which the beam uses up all its strength carrying nothing but itself. The code below works it out for a 5,000 mm² steel section with a moment of area of 8.0×10^7 mm⁴ and a working stress limit of 165 MPa (roughly the 355 MPa steel of How Bridges Work 1 with a factor of safety a little over 2):

```
import math

RHO, G = 7850, 9.81                     # steel density, gravity
area, i, y = 5000, 8.0e7, 150           # mm2, mm4, mm
allow = 165                             # MPa, working stress

w = RHO * G * area * 1e-6 / 1000        # own weight in kN/m
print(round(w, 3))                      # 0.385
for span in (10, 20, 40, 80):
    print(span, round(w * span**2 / 8, 1))
# 10 4.8
# 20 19.3
# 40 77.0
# 80 308.0

m_cap = allow * (i / y) / 1e6           # moment the section can carry, kN m
print(round(m_cap, 1), round(math.sqrt(8 * m_cap / w), 1))
# 88.0 42.8
```

The beam weighs 0.385 kN per metre. Its own-weight moment is only 4.8 kN·m at 10 m, but 77 kN·m at 40 m and 308 kN·m at 80 m: each doubling quadruples it. The section can carry 88 kN·m, so it breaks under its own weight at 42.8 m, *before* a single vehicle drives on. In practice

the limit is far shorter, because the live load takes much of the capacity (Exercise 3 shows this). Making the beam deeper helps, but not as much as you would like, because a deeper beam is also a heavier one.

That is why beam bridges are used for short and medium spans (the approach spans, the viaducts, the footbridges), and why every longer crossing uses another idea. An **arch** turns bending into compression (How Bridges Work 3). A **truss** builds a deep beam out of light struts and ties (How Bridges Work 4). A **cable** carries load in pure tension, with no bending at all (How Bridges Work 7 and 8). The Forth bridges are three different answers to a span that a plain beam could never reach.

> Common mistakes
> Mixed-up units, above all: `I` in mm^4 needs the moment in N·mm (kN·m × 10^6) and `y` in mm; putting `b` and `h` the wrong way round in `b×h³/12` (the depth is the cubed one); using the moment formula for a point load on a uniform-load problem; and forgetting that the beam's own weight is part of the load.

Where This Leads

A beam carries load by bending, and bending is the least efficient way to use material: only the outer faces are working hard. The next chapter takes the oldest answer to that problem, the stone arch, where the whole shape is squeezed in compression, and looks at the Roman arches, the clapper bridges and the packhorse bridges of Scotland.

EXERCISE 1

A timber joist spans 4 m and carries a uniform load of 6 kN/m including its own weight. Its section is 75 mm × 225 mm. Write a program that works out the maximum moment and shear, then the maximum bending stress when the joist is on edge (225 mm deep) and when it is laid flat (75 mm deep). By what factor does the stress change, and why is it 3 when the moment of area changes by a factor of 9?

View solution

EXERCISE 2

A steel beam spans 6 m and carries one 80 kN load at midspan (ignore its own weight). The stress limit is 165 MPa and the deflection limit is span/360; E = 200,000 MPa. For a point load at midspan, the sag

is `P×L³/(48×E×I)`. Test these I-sections (flange width, flange thickness, web thickness, depth in mm): (100, 8, 5, 200), (125, 10, 6, 250), (150, 12, 8, 300), (150, 14, 8, 350). For each, print the stress, the sag and whether it passes both limits. Predict first which is the smallest that passes.

View solution

EXERCISE 3

Take I-sections with 150 × 12 mm flanges and an 8 mm web, at depths of 300, 450 and 600 mm. For each, find the moment capacity at 165 MPa, the beam's own weight per metre, and the longest simply supported span (a) when it carries only its own weight and (b) when it carries a further 10 kN/m. By how much does doubling the depth increase each span, and what does that tell you about the limits of a beam?

View solution

QUICK REFERENCE

- **Uniform load:** max shear wL/2 at the supports; max moment $wL^2/8$ at midspan
- **Midspan point load:** max shear P/2; max moment PL/4
- **Bending stress:** M × y / I, largest at the outer faces; zero at the neutral axis
- **Rectangle:** $I = b \times h^3/12$; depth counts cubed, width once
- **I-beam:** flanges far from the neutral axis resist bending; the web resists shear
- **Deflection (uniform load):** $5wL^4/(384EI)$; steel E ≈ 200,000 MPa
- **Span:** moment grows with L^2 and sag with L^4, so a plain beam only suits short spans

COMING NEXT

How Bridges Work 3 turns to arches and early bridges: the Roman arch, thrust and abutments, stone and brick construction, and the clapper and packhorse bridges of Scotland.

How Bridges Work

Chapter 3 · Arches and Early Bridges

How Bridges Work 2 ended with a problem: a beam carries load by bending, and bending wastes material. Stone makes it worse, because stone is strong when squeezed and weak when stretched, and bending always stretches one face. Builders who had only stone, brick and timber found a way round that, and it lasted two thousand years: shape the structure so that the load squeezes every part of it. That shape is the **arch**. This chapter looks at the simplest stone bridge (a slab), the arch that replaced it, the sideways push that every arch creates, and the Scottish bridges that show all of these at work.

> **WHAT WAS RUN**
>
> Every Python snippet in this chapter and in the three exercise solutions was executed, and the output shown in the comments is what Python printed. Material strengths and some dimensions are illustrative round figures, and are marked where they appear.

Stone Likes to Be Squeezed

Materials differ in which force they prefer (How Bridges Work 1). Stone, brick and mass concrete are all **strong in compression and weak in tension**: a block of stone will carry an enormous squeezing load, but a pull of only a small fraction of that tears it apart. So a stone structure must be shaped to avoid tension. A beam cannot do that, since bending puts the underside in tension. An arch can, because its curve turns the vertical load into a squeeze along the ring of stones.

The Clapper Bridge: A Stone Beam

The simplest stone bridge is a flat slab laid from bank to bank, or from pier to pier, which is a beam with all the problems of How Bridges Work 2. These are called **clapper bridges**, and Scotland has many of them. Historic Environment Scotland describes Achnamara clapper bridge in Argyll as the last survivor of a group of mid-Argyll clapper bridges, and the national record (Canmore) classes it as 17th century. The slab fails when the tension on its underside reaches what the stone can take. The code finds the longest span for a slab 1 m wide, carrying only its

own weight, using an *illustrative* working tension of 3 MPa (real stone varies widely, and a single flaw can break a slab):

```
import math

RHO, G = 2600, 9.81          # stone density kg/m3, gravity
ALLOW = 3.0                  # MPa, illustrative working tension for stone

def slab_span(thick_m, width_m=1.0):
    w = RHO * G * thick_m * width_m / 1000          # kN/m, own weight
    z = width_m * 1000 * (thick_m * 1000) ** 2 / 6  # section modulus, mm3
    m_cap = ALLOW * z / 1e6                         # kN m
    return w, m_cap, math.sqrt(8 * m_cap / w)

for t in (0.2, 0.3, 0.4):
    w, m, s = slab_span(t)
    print(t, round(w, 2), round(m, 1), round(s, 1))
# 0.2 5.1 20.0 5.6
# 0.3 7.65 45.0 6.9
# 0.4 10.2 80.0 7.9
```

The columns are the thickness (m), the slab's own weight (kN/m), the moment it can carry (kN·m) and the longest span (m). A 0.2 m slab reaches only 5.6 m *with nothing on it*, and doubling the thickness gains only about 40% (7.9 m), because the span grows with the square root of the thickness and the slab gets heavier as it gets thicker. A clapper bridge therefore crosses wide water with several slabs resting on stone piers in the stream, never with one long slab.

How an Arch Works

An arch is a ring of wedge-shaped stones called **voussoirs** (say "voo-SWAHR"). The one at the top is the **keystone**. Each stone is squeezed between its neighbours, and the load travels around the curve to the foundations at each end. Because each joint is squeezed shut, the arch needs little or no tension capacity, and the stones are held in place by the squeeze rather than by the mortar between them. Roman builders used the **semicircular arch** throughout their empire. The

Pont du Gard, the Roman aqueduct bridge in southern France, stacks three tiers of arches, and its large stones are laid without mortar.

There is one practical catch. Until the keystone is in, the arch cannot carry anything: each half would simply fall inward. Builders therefore built the arch over a temporary timber frame called **centring**, laid the voussoirs on it, set the keystone, and only then removed the frame. The arch then carried itself.

Thrust: The Hidden Sideways Push

The curve of an arch does not only press down. It also pushes *outwards* at the two ends (the **springings**), like the sides of a bowed bow trying to spring open. This sideways push is called **thrust**, and it is the key fact about every arch. For a parabolic arch under a uniform load `w` across a span `L`, with a rise (height from springing to crown) of `f`, the horizontal thrust is `H = wL²/(8f)`. The vertical reaction is the same as for a beam: `wL/2`.

```
def thrust(w, span, rise):
    """Horizontal thrust (kN) of a parabolic arch under a uniform load w (kN/m)."""
    return w * span**2 / (8 * rise)

w, span = 50, 12
v = w * span / 2                        # vertical reaction at each springing
print(v)                                # 300.0
for rise in (1.5, 3, 6):
    h = thrust(w, span, rise)
    print(rise, h, round((h**2 + v**2) ** 0.5, 1))
# 1.5 600.0 670.8
# 3 300.0 424.3
# 6 150.0 335.4
print(w * span**2 / 8)                  # beam moment for comparison: 900.0
```

The columns are the rise (m), the thrust (kN) and the resultant force on the springing (kN). The vertical reaction is 300 kN whatever the shape. But the thrust is inversely proportional to the rise: **flatten the arch to a quarter of the height and the thrust quadruples**, from 150 kN to 600 kN. The last line shows what the arch has done to How Bridges Work 2's problem: the beam with the same span and load would have a bending moment of 900 kN·m, and the arch replaces it

with a thrust times a lever arm (the rise), since 600 × 1.5 = 900. The arch trades bending for a horizontal push that the ground must resist.

Abutments: Holding the Push

The ends of an arch rest on **abutments** (end supports) or **piers** (supports between arches), and these must resist the thrust without sliding sideways or tipping over. A heavy block does both: weight gives friction against sliding, and weight gives a restoring moment against tipping. The code tests a masonry abutment block 4 m high, per metre length of the bridge, with a vertical load of 300 kN and a thrust of 150 kN acting at the top. The friction coefficient of 0.6 and the density of 2200 kg/m³ are illustrative:

```
G, RHO, MU = 9.81, 2200, 0.6     # gravity, masonry density, friction

def abutment(b, height, v, h_thrust, h_arm):
    """Safety factors for a block b m wide, per metre run of bridge."""
    wt = RHO * G * b * height / 1000          # kN/m, block weight
    sliding = MU * (wt + v) / h_thrust
    overturn = (wt + v) * b / 2 / (h_thrust * h_arm)   # moments about the outer toe
    return round(sliding, 2), round(overturn, 2)

print(abutment(2.0, 4, 300, 150, 4))   # (1.89, 0.79)
print(abutment(3.0, 4, 300, 150, 4))   # (2.24, 1.4)
print(abutment(4.0, 4, 300, 150, 4))   # (2.58, 2.15)
```

A factor below 1 means the block moves. The 2 m block has a comfortable sliding factor (1.89) but an overturning factor of only 0.79, so it would tip. At 3 m it is still short (1.4), and at 4 m both pass (2.58 and 2.15). Notice that **tipping, not sliding, sets the width**. This simple model, with the loads through the middle of the block, is only a first check: real masonry design also asks where the line of thrust passes through the stones.

> Where the line of thrust goes
> In a real masonry arch the load path follows a curve called the **line of thrust**. If it stays inside the ring of stones, every joint is in compression and the arch stands. If it escapes the ring, a joint opens, and a few such openings turn the arch into a mechanism that collapses. This is why masonry arches fail from

moving supports or heavy point loads rather than from "crushed stone", and why abutments that settle or spread are dangerous.

Early Bridges in Scotland

Scotland's old bridges show every idea in this chapter. A few examples, with what the search I made for this chapter supports and what comes from general knowledge:

BRIDGE	TYPE	WHAT IT SHOWS
Achnamara (Argyll)	clapper (stone slabs)	classed as 17th century by Canmore; the last survivor of a group of mid-Argyll clapper bridges
Brig o' Balgownie (Aberdeen)	single arch	reported as the earliest surviving single-arch bridge in Scotland, about 1320
Old Stirling Bridge	multi-arch	reported as the oldest surviving multi-arch bridge in Scotland, from the 1400s or 1500s; it crosses the River Forth, upstream of the three bridges this course follows
Carrbridge packhorse bridge	single arch	over the River Dulnain, finished in 1717; the great flood of 1829 (the "Muckle Spate") swept away its parapets and it survived
Tay Bridge, Aberfeldy	five arches	one of General Wade's Highland bridges, designed by William Adam (general knowledge: it dates from the 1730s)

A **packhorse bridge** like Carrbridge is deliberately narrow, with low parapets so that the loaded baskets (panniers) on a horse's flanks could pass, and it is often steeply humped; the formula above shows the benefit of a high rise, since the taller the arch, the smaller the thrust the abutments must resist. The Carrbridge survival in 1829 is a nice example of the arch's strength: a flood that destroyed many bridges broke the parapets off this one and left the arch standing. The same strength has a weakness: arches resist floods and loads well but dread movement of their abutments and piers, a problem that comes back in How Bridges Work 9, when we look at scour and failure.

Where This Leads

The arch solved the problem of stone, but it needs a large mass of masonry to hold the thrust, and it is limited to the span a ring of stones can safely reach. The next idea is a clever one: instead of curving a heavy ring, build a light, deep frame out of straight members arranged in triangles, so that each piece carries only tension or only compression. That is the truss, the subject of How Bridges Work 4.

EXERCISE 1

A clapper slab is 1 m wide and may be 0.2, 0.3 or 0.4 m thick. Stone density is 2600 kg/m^3. For working tensions of 1, 2 and 3 MPa (all illustrative), find the longest span (a) when the slab carries only its own weight and (b) when it also carries 3 kN/m of people and animals. Print a table and explain what happens when the thickness doubles.

View solution

EXERCISE 2

A parabolic arch of span 20 m carries a uniform load of 40 kN/m. For rises of 2, 4, 5 and 10 m, print the vertical reaction, the horizontal thrust and the resultant force, and check that the beam moment $wL^2/8$ equals the thrust times the rise. Then work out the smallest rise that keeps the thrust at or below 400 kN. What fraction of the span is it?

View solution

EXERCISE 3

An arch pushes on a masonry abutment block 4 m high with a vertical force of 300 kN and a horizontal thrust acting at the top. Density is 2200 kg/m^3 and the friction coefficient is 0.6. Write a loop that finds the narrowest block (in 0.1 m steps from 1 m) with a sliding factor of at least 1.5 and an overturning factor of at least 2.0, for a thrust of 150 kN and then 300 kN. Which check decides the width, and what does the result say about flat arches?

View solution

QUICK REFERENCE

- **Stone, brick, mass concrete:** strong in compression, weak in tension

- **Clapper bridge:** a stone slab acting as a beam; short spans only
- **Arch:** a ring of voussoirs with a keystone; the load travels around the curve as compression
- **Centring:** the temporary frame that carries an arch until the keystone is in
- **Thrust:** $H = wL^2/(8f)$ for a parabolic arch; flatter means more thrust
- **Beam moment = thrust × rise:** the arch swaps bending for a horizontal push
- **Abutments:** must resist sliding (friction) and tipping (weight); tipping often decides

COMING NEXT

How Bridges Work 4 turns to trusses: why triangles make a rigid frame, the Warren and Pratt layouts, which members are in tension and which in compression, and the method of joints for finding the force in each.

How Bridges Work

Chapter 4 · Trusses

How Bridges Work 2 showed that a deep beam is efficient because its material sits far from the neutral axis, and that an I-beam throws away the useless middle. A **truss** takes that idea to its limit. It replaces the solid web with a skeleton of straight bars, and arranges the bars in triangles so that every bar carries a simple pull or a simple push and nothing else. The result is a deep, light beam that can be made from timber, iron or steel, and that was widely used for railway bridges in the nineteenth century. This chapter explains why triangles work, names the classic layouts, and shows how to work out the force in every bar with a short program.

> **WHAT WAS RUN**
>
> Every Python snippet in this chapter and in the three exercise solutions was executed, and the output shown in the comments is what Python printed. The truss solver uses only the standard library.

Why Triangles

Take four bars and join them at their ends with pins into a rectangle. Push on one corner and the rectangle collapses sideways into a parallelogram. Take three bars and pin them into a triangle, and nothing you do to the corners will change its shape without bending or breaking a bar. **A triangle is the only polygon that is rigid on its own.** Add a diagonal to the rectangle and it becomes two triangles, and it is rigid too. Every truss is a row of triangles.

In the model engineers use, the bars are joined at **joints** by frictionless pins, and the loads act only at the joints. Then each bar is pulled or pushed along its own length and does not bend: it is in **tension** (stretched) or **compression** (squeezed), the two simple forces of How Bridges Work 1. Real trusses have riveted, bolted or welded joints, but the pin model gives the right forces to a good approximation. For a plane truss with `j` joints and `m` bars, the structure is just rigid, with no spare bars, when `m + 3 = 2j` (the 3 stands for the support reactions). Fewer bars and it can collapse; more and it needs extra analysis.

The Parts of a Truss

PART	WHERE	USUALLY CARRIES	DOES THE JOB OF
Top chord	the upper horizontal members	compression (in a span with the deck loaded from above)	the top flange of an I-beam
Bottom chord	the lower horizontal members	tension	the bottom flange
Diagonals and verticals	between the chords	tension or compression, depending on layout	the web: they carry the shear
Joints	where members meet	no bending in the model	pins

This table matches How Bridges Work 2. Under a downward load the whole truss sags like a beam: the top is squeezed, the bottom is stretched, and the diagonals and verticals carry the shear between them.

Three Classic Layouts

Nineteenth-century engineers patented several arrangements. Here are the three you will meet most often, with the facts that the sources I checked agree on:

LAYOUT	PATENT	DIAGONALS (UNDER GRAVITY LOAD)	VERTICALS
Howe	1840, William Howe	slope towards the centre; in compression	in tension
Pratt	1844, Thomas and Caleb Pratt	the reverse of the Howe; in tension	in compression
Warren	1848, England (credited to James Warren and Willoughby Monzani)	alternate between tension and compression	none required

(The sources differ on some details: whether the Pratt patent names Thomas alone or both Thomas and Caleb, how the second Warren name is spelt, and the patent drawing is said to differ from the form later called a Warren truss.) The Pratt was a clever match of materials and forces. Its long diagonals are in tension and its short verticals in compression. A compression bar can *buckle* (see below), so you want compression bars short; a tension bar cannot buckle, so it can be long and thin. That is why the earliest Pratt trusses paired wooden compression members

with iron tension rods. The Warren needs fewer bars and fewer joints, and its simplicity has kept it in use ever since.

```
a small Warren truss (supports at A and C, load at B)

    D─────E
   / \   / \        top chord: D to E
  /   \ /   \
 A─────B─────C      bottom chord: A to B to C
       ↓
      load
```

The Method of Joints

To find the force in every bar, treat each joint as a small object in balance. At every joint the forces must add up to zero in two directions, horizontal and vertical. Each bar pulls on the joint along its own line, so a bar's force can be split into horizontal and vertical parts using its direction. For a truss with `j` joints there are `2j` equations, exactly enough for the `m` bar forces and 3 reactions. Doing them by hand takes pencil and patience; a computer can set them all up and solve them at once.

The program below does exactly that. It takes the joint positions, the list of bars, the loads and the supports, and writes the `2j` equations into a table (a matrix). It then solves them by Gaussian elimination, the textbook way of solving simultaneous equations. A positive answer means a bar in tension; a negative one means compression.

```
import math

def solve_truss(joints, members, loads, supports):
    """Method of joints as one linear system. Tension is positive."""
    names = list(joints)
    n = len(members) + len(supports)
    rows = {}
    for j in names:
        rows[(j, "x")] = [0.0] * n
        rows[(j, "y")] = [0.0] * n
```

```
    for k, (a, b) in enumerate(members):
        (xa, ya), (xb, yb) = joints[a], joints[b]
        length = math.hypot(xb - xa, yb - ya)
        cx, cy = (xb - xa) / length, (yb - ya) / length
        rows[(a, "x")][k] += cx
        rows[(a, "y")][k] += cy
        rows[(b, "x")][k] -= cx
        rows[(b, "y")][k] -= cy
    for k, (j, d) in enumerate(supports):
        rows[(j, d)][len(members) + k] = 1.0
    keys = list(rows)
    a = [rows[key] + [-(loads.get(key[0], (0, 0))[0 if key[1] == "x" else 1])] for key i
    for c in range(n):                                  # Gaussian elimination
        p = max(range(c, n), key=lambda r: abs(a[r][c]))
        a[c], a[p] = a[p], a[c]
        for r in range(n):
            if r != c:
                f = a[r][c] / a[c][c]
                a[r] = [x - f * y for x, y in zip(a[r], a[c])]
    return [a[i][n] / a[i][i] for i in range(n)]

joints = {"A": (0, 0), "B": (4, 0), "C": (8, 0), "D": (2, 3), "E": (6, 3)}
members = [("A", "B"), ("B", "C"), ("D", "E"), ("A", "D"), ("D", "B"), ("B", "E"), ("E",
loads = {"B": (0, -40)}
supports = [("A", "x"), ("A", "y"), ("C", "y")]
res = solve_truss(joints, members, loads, supports)
for (a, b), f in zip(members, res):
    print(a + b, round(f, 2), "T" if f > 1e-9 else ("C" if f < -1e-9 else "-"))
print([round(r, 2) for r in res[len(members):]])
# AB 13.33 T
# BC 13.33 T
# DE -26.67 C
# AD -24.04 C
# DB 24.04 T
# BE 24.04 T
# EC -24.04 C
# [0.0, 20.0, 20.0]
```

The last line gives the three reactions: no horizontal reaction at A, and 20 kN up at each support, which is half the 40 kN load each (the load is in the middle). Read the table: the bottom chord (AB and BC) is in tension at 13.33 kN, the top chord (DE) in compression at 26.67 kN, and the outer diagonals AD and EC in compression, with the two diagonals that meet at the loaded joint,

DB and BE, in tension. **You can check one by hand.** At joint A the 20 kN reaction must be balanced by the vertical part of AD: AD points up and to the right at a slope of 3 in 2, so 20 + F×(3/3.606) = 0, giving F = −24.04 kN (compression). The program agrees. It is worth doing that check on every program you write: a single hand-verified value catches a sign error that would otherwise corrupt every answer.

Deeper Is Lighter

The top and bottom chords carry the beam's bending moment, just as the flanges of an I-beam do (How Bridges Work 2): the force in a chord is roughly the beam moment divided by the depth of the truss, `F = M / depth`. The code below applies that to a 40 m truss under a uniform load of 20 kN/m:

```
w, span = 20, 40                    # kN/m uniform load, m
m = w * span**2 / 8                 # beam moment, kN m
print(m)                            # 4000.0
for depth in (4, 5, 8, 10):
    print(depth, m / depth)         # chord force = M / depth
# 4 1000.0
# 5 800.0
# 8 500.0
# 10 400.0
```

The beam moment is 4,000 kN·m. A truss 4 m deep needs chords carrying 1,000 kN; at 10 m deep each chord carries only 400 kN. A deeper truss has lighter chords, but longer diagonals and verticals, and more of them, and a deeper structure is harder to build and to keep stiff against the wind. Truss depth is therefore a compromise.

Buckling: Why Compression Bars Are Dangerous

A tension bar fails only when the material tears. A compression bar can fail much earlier, by suddenly bowing sideways, called **buckling**. Try pushing the two ends of a plastic ruler together: it bows out long before it crushes. The load at which a slender, pin-ended strut buckles was worked out by Euler: `P = π²EI / L²`, where `E` is the stiffness of the material, `I` is the same

second moment of area as in How Bridges Work 2 (for the weakest direction of the cross-section), and `L` is the length.

```
import math

E = 200e3                              # MPa

def euler_kn(i_mm4, length_m):
    """Buckling load (kN) of a pin-ended strut."""
    return math.pi**2 * E * i_mm4 / (length_m * 1000) ** 2 / 1000

i = 8.0e6                              # mm4, a modest tube or angle
for length in (2, 4, 8):
    print(length, round(euler_kn(i, length)))
# 2 3948
# 4 987
# 8 247
```

Doubling the length divides the buckling load by **four**: a 2 m strut carries 3,948 kN, a 4 m strut 987 kN, an 8 m strut only 247 kN. This is the reason compression members are kept short, braced against the sideways movement, or made as hollow tubes (which have a large `I` for little material). The great cantilever arms of the Forth Bridge are a famous example: its big compression members are steel tubes, and its tension members are lattice (this is general knowledge, and How Bridges Work 6 looks at it properly).

> Common mistakes
> Using the pin model on a truss with a long beam-like member that carries load between joints (it bends, and the model is wrong); taking a negative answer as an error instead of reading it as compression; and checking a compression bar only for crushing. Always check buckling, which usually fails first.

Where This Leads

Trusses give a light, deep beam for spans up to a few hundred metres, but all that steel still has to be made, delivered and kept free from rust. The next chapter steps back from shapes to the

stuff bridges are made of: timber, stone, cast and wrought iron, steel and concrete, and why the Forth Bridge is painted.

EXERCISE 1

Using the solver and the truss of this chapter (joints A, B, C at x = 0, 4, 8 m on the bottom and D, E at (2, 3) and (6, 3) on top), solve it for (a) 60 kN downward at the top joint E and (b) 40 kN at B and 60 kN at E together. Print the force in each bar, whether it is in tension or compression, and the reactions, and check that the vertical reactions add up to the load. Which bar changes from compression to tension between (a) and (b)?

View solution

EXERCISE 2

Build a Pratt truss with joints A to E on the bottom chord at x = 0, 4, 8, 12, 16 m and F, G, H on top at (4, 3), (8, 3), (12, 3). Members: bottom AB, BC, CD, DE; top FG, GH; end posts AF and HE; verticals BF, CG, DH; diagonals FC and HC. A pin at A and a roller at E carry 50 kN down at each of F, G and H. Check that `m + 3 = 2j`, solve it, and say which bars are in tension, which in compression, and which carry no force.

View solution

EXERCISE 3

A 40 m truss carries 20 kN/m. Its top chord is a pin-ended strut with $I = 8.0\times10^{6}$ mm^4 and a panel length of 4 m (E = 200,000 MPa). For depths of 4, 5, 8 and 10 m work out the chord force (moment divided by depth) and say whether the strut passes with a factor of safety of 2 against buckling. Then find the smallest depth, in 0.5 m steps, that does.

View solution

QUICK REFERENCE

- **Triangle:** the only rigid polygon; every truss is built from triangles
- **Pin model:** loads at joints only, so each bar is purely tension or compression
- **Rigidity count:** m + 3 = 2j for a plane truss with no spare bars

- **Chords:** the top is usually in compression, the bottom in tension; chord force ≈ M / depth
- **Pratt:** diagonals in tension, verticals in compression; **Howe:** the reverse; **Warren:** alternating diagonals
- **Method of joints:** forces balance at every joint, giving 2j equations; positive is tension
- **Buckling:** $P = \pi^2 EI/L^2$; doubling the length cuts it to a quarter

COMING NEXT

How Bridges Work 5 is about materials: timber, stone, cast and wrought iron, steel and concrete, how fatigue and corrosion wear a bridge down, and why the Forth Bridge is painted.

How Bridges Work

Chapter 5 · Materials

A bridge's shape tells you which forces it uses (beam, arch, truss), but the material decides whether it can survive them. The history of bridges is as much a history of materials as of ideas: timber and stone, then cast iron, wrought iron and steel, then concrete. Each was better than the last at one job and worse at another, and each failed in its own characteristic way. This chapter compares them with a simple measure of strength for weight, then looks at the two slow enemies that no bridge escapes, **fatigue** and **corrosion**, and at what the Forth Bridge's famous paint is really for.

WHAT WAS RUN, AND WHAT TO TRUST

Every Python snippet in this chapter and in the three exercise solutions was executed, and the output shown in the comments is what Python printed. The material strengths in the code are **rounded, illustrative figures** (real values vary with grade, age and condition), so use the table to see the pattern, not as design data.

The Six Materials

MATERIAL	LIKES	DISLIKES	IN BRIDGES
Timber	light; good in tension and compression along the grain	rot, insects, fire; weak across the grain	early beams and trusses, temporary works
Stone and brick	compression; durable	tension	arches, piers, abutments
Cast iron	compression; can be cast into shapes	tension and shock; brittle	early iron arches and columns
Wrought iron	tension; ductile, less brittle than cast iron	costly to make in large pieces; slag inside	mid-1800s girders, tubes and trusses
Steel	tension and compression; strong, ductile	rust; fatigue; fire	almost every large modern bridge
Concrete	compression; mouldable, cheap	tension (cracks)	piers, decks, towers; reinforced or prestressed

The most useful fact in this table is the split between tension and compression, which How Bridges Work 3 already used: stone, brick and plain concrete are almost useless when pulled, and cast iron is nearly as bad. A bridge made of them has to be built so that the load squeezes them. A material that is good in tension (wrought iron, steel, and timber along its grain) can be used in a cable, a hanger or a tie.

Strength for Weight

A bridge has to carry itself before it carries anything else (How Bridges Work 1). A fair measure of a material is therefore how long a uniform bar of it can be before it just carries its own weight: `length = strength / (density × g)`. The longer the length, the less of a material's strength is wasted just holding itself up. Here it is worked out for the six materials, in tension and in compression, with rounded illustrative values:

```
G = 9.81
# name: (density kg/m3, tension MPa, compression MPa): rounded, illustrative
MATS = {
    "timber (along grain)": (500, 20, 25),
    "granite":              (2700, 8, 150),
    "plain concrete":       (2400, 3, 40),
    "cast iron":            (7200, 150, 600),
    "wrought iron":         (7700, 250, 250),
    "structural steel":     (7850, 355, 355),
}

def length_km(mpa, rho):
    """Length of a uniform bar that just carries its own weight, in km."""
    return mpa * 1e6 / (rho * G) / 1000

for name, (rho, t, c) in MATS.items():
    print(f"{name:22} {length_km(t, rho):6.2f} {length_km(c, rho):6.2f}")
# timber (along grain)     4.08   5.10
# granite                  0.30   5.66
# plain concrete           0.13   1.70
# cast iron                2.12   8.49
# wrought iron             3.31   3.31
# structural steel         4.61   4.61
```

The two columns are the length in kilometres in tension and in compression. Read them like this. **Steel** is the best in tension (4.61 km), which is why every cable-supported bridge uses it. **Granite** has a compression length of 5.66 km but a tension length of only 0.30 km, nineteen times less: stone is a perfect column and a useless cable. **Plain concrete** manages only 0.13 km in tension, the weakest of all. **Timber** is a surprise: at about 4 km it is almost as good as steel for its weight, which is why wooden structures can be so light, although it rots and burns. **Cast iron** is the champion in compression (8.49 km) and poor in tension (2.12 km), which is exactly the weakness that catches it out. Real design adds buckling, joints and a safety factor, so treat these as comparisons, not predictions.

A Short Tour: From Timber to Steel

Timber was the first bridge material and is still used. It is light and easy to work, and a good truss built from it needs no great skill (the early Pratt trusses of How Bridges Work 4 paired wooden compression members with iron tension rods). Its enemies are rot and insects.

Stone and brick gave the long-lived arches of How Bridges Work 3. They last for centuries, but only while the load keeps them squeezed.

Cast iron was the first new bridge material of the industrial age. The Iron Bridge at Coalbrookdale in Shropshire (about 1779) is the famous early example, an arch of cast-iron ribs (general knowledge, not checked for this chapter). Cast iron is strong when squeezed, which suits an arch, but it is brittle when pulled, and it breaks without warning. That weakness features in a famous disaster: the first Tay Bridge collapsed in December 1879 with a train on it, and investigators found that the cast-iron lugs that held parts of the structure together were the critical weak point; in tests for the inquiry, lugs broke at only about 20 long tons, rather than the 60 expected. Some accounts add that fatigue cracks had formed in the lugs before the disaster, and the inquiry blamed faults of design, construction and maintenance.

Wrought iron is tougher: it stretches and deforms before it fails, so it gives a warning. In the 1840s Robert Stephenson used it for the great box girders of the Britannia Bridge over the Menai Strait (How Bridges Work 2), and for many railway girders and trusses.

Steel combines the best of both: strong in tension and compression, and ductile. Once it could be made cheaply and in quantity, it took over the long-span bridges, and the Forth Bridge of 1890, the subject of How Bridges Work 6, was built of it (general knowledge).

Concrete is the modern stone. It is poured to any shape, but plain concrete is weak in tension (see the table). Two cures exist: put steel bars inside where tension will occur (**reinforced** concrete), or squeeze the concrete in advance with stretched steel cables so that the load never puts it in tension (**prestressed** concrete). Both are standard techniques.

Fatigue: Death by a Thousand Cycles

A steel bar that holds a steady load for ever may fail if the load is applied and removed over and over, even when each cycle is well below the strength. This is **fatigue**: tiny cracks start at a notch, a weld toe or a rivet hole and grow a little with every cycle until the piece suddenly breaks. What matters is the *range* of stress in each cycle and the number of cycles. A widely used simplified model says the number of cycles to failure falls with the *cube* of the stress range. The code uses a "detail category" of 71 MPa, meaning that a stress range of 71 MPa lasts 2 million cycles; the category and the model are simplified and illustrative:

```
def cycles_to_fail(delta_sigma, category=71):
    """Simplified S-N curve: N = 2e6 * (category / range)^3."""
    return 2e6 * (category / delta_sigma) ** 3

for s in (35, 71, 142):
    print(s, round(cycles_to_fail(s)))
# 35 16695557
# 71 2000000
# 142 250000

trains_per_day = 150
n = cycles_to_fail(45)
print(round(n), round(n / (trains_per_day * 365), 1))
# 7855385 143.5
```

Halving the stress range multiplies the life by eight, and doubling it divides the life by eight (16.7 million cycles at 35 MPa, 250,000 at 142 MPa). At 150 trains a day, each causing one 45 MPa cycle, the detail would last about 143.5 years. That is why engineers spend effort on the details of a bridge: a welded or riveted joint with a poor shape raises the local stress range and cuts the life sharply, and why a small increase in axle load matters more than it looks (Exercise 2).

Corrosion: Rust Never Sleeps

Steel rusts when it meets oxygen and water, and rust takes material from the section. A thinner plate carries the same force at a higher stress. The code below follows a plate 200 mm wide and 12 mm thick, carrying a steady 240 kN, that loses 0.05 mm from each face every year (an assumed rate; real rates depend on the climate and the sheltering of the member):

```
width, thick0, force = 200, 12.0, 240e3        # mm, mm, N
rate = 0.05                                    # mm per year lost from each face (assume

def stress(years):
    thick = thick0 - 2 * rate * years
    return force / (width * thick)

for years in (0, 20, 40, 60, 80):
    print(years, round(stress(years), 1))
# 0 100.0
# 20 120.0
# 40 150.0
# 60 200.0
# 80 300.0

limit = 165                                    # MPa
t_min = force / (width * limit)
print(round(t_min, 2), round((thick0 - t_min) / (2 * rate), 1))
# 7.27 47.3
```

The stress climbs slowly and then faster, because losing a millimetre from a thin plate matters more than from a thick one: it is 100 MPa new, 150 MPa after 40 years, and 300 MPa after 80, which is already beyond the limit. The plate reaches the 165 MPa limit after 47.3 years. Corrosion rarely strikes evenly, either. Pits, crevices and the places where water collects (joints, drainage points, the underside of a deck) wear much faster than the average, so inspectors look at those first.

Why the Forth Bridge Is Painted

Paint is the cheapest protection against corrosion: it keeps water and air off the steel. The Forth Bridge, with a huge area of exposed steel and a salt-laden sea climate, needs protection on a vast scale. The saying "painting the Forth Bridge" came to mean a task that is never finished, the idea being that as soon as one end had been painted, it was time to start again at the other. That ended in 2011: Network Rail announced on 9 December 2011 that a ten-year, £130 million refurbishment was complete, ahead of schedule, and the bridge was free of scaffolding for the first time in a decade.

The new coating was a **glass flake epoxy**, a type used in the offshore oil industry, designed to last about 25 years (one trade report quotes 30). One source gives the layers as a zinc-based primer of 35 microns, an intermediate coat of glass flake epoxy 400 microns thick, and a polyurethane gloss top coat of 35 microns, applied over an area of more than 230,000 square metres. The glass flakes make a layered barrier that water finds hard to cross. The zinc primer is a standard feature of steel protection: zinc corrodes in preference to the steel beneath it, so the primer protects even where the coat above has been scratched (general knowledge). The numbers in this paragraph come from one source each and have not been cross-checked.

> Common mistakes
> Using one strength figure for tension and compression (the two are very different for stone, concrete and cast iron); judging a bridge only by its peak load and forgetting the number of cycles; treating a corrosion rate as a fixed constant (it depends on climate, drainage and coating); and assuming a coating lasts for ever (it is a maintenance cycle, not a one-off).

Where This Leads

You now have the toolkit for forces (How Bridges Work 1), beams (2), arches (3), trusses (4) and materials (5). The next chapter turns to the structure that has made the Forth famous: the cantilever bridge, a design that balances huge steel arms over their piers, and the story of the engineers who built it in 1890.

EXERCISE 1

Using the table of materials in this chapter and adding brick (1900 kg/m^3, 1 MPa in tension, 20 MPa in compression), work out the self-weight length in tension and compression for each material, print the table with formatted columns, then rank the materials for a hanging cable (tension) and for a column (compression). Which materials appear near the top of both rankings, and which only in one?

View solution

EXERCISE 2

A bridge detail has fatigue category 71 MPa (N = 2×10^6 cycles at a range of 71 MPa, with N proportional to 1 / range^3). Every day 100 light trains cause a range of 45 MPa and 10 heavy trains cause 70 MPa, one cycle each. Using Miner's rule (damage = cycles applied ÷ cycles to failure, summed over the loads, with failure at 1), find the life in years, and then the life if the heavy trains double to 20 a day. How much does one heavy cycle cost compared with one light cycle?

View solution

EXERCISE 3

A 200 mm × 12 mm plate carries a steady 240 kN and loses material from both faces at 0.01, 0.05 or 0.1 mm per face per year (all assumed). The allowable stress is 165 MPa. For each rate find how many years it takes to reach that stress and the stress after 50 years. Then suppose a coating stops corrosion completely for the first 25 years: when does the plate reach the limit for each rate?

View solution

QUICK REFERENCE

- **Stone, brick, plain concrete, cast iron:** far stronger in compression than in tension
- **Wrought iron and steel:** good in tension; steel is best for cables and ties
- **Timber:** light and efficient along the grain; rots and burns
- **Reinforced and prestressed concrete:** steel bars or stretched cables supply the tension
- **Self-weight length:** strength ÷ (density × g); a fair measure of strength for weight
- **Fatigue:** repeated cycles crack a detail; life falls with the cube of the stress range (simplified)
- **Corrosion:** thins the section, so the stress rises; coatings buy years but need renewing

COMING NEXT

How Bridges Work 6 is about cantilevers and the Forth Bridge: the balanced cantilever principle, the 1890 Forth Bridge and its engineers Fowler and Baker, and the famous human-model demonstration.

How Bridges Work

Chapter 6 · Cantilevers and the Forth Bridge

This is the chapter the whole course has been walking towards. The Forth Bridge, opened on 4 March 1890 by the Duke of Rothesay (the future Edward VII), is designed by Sir John Fowler and Sir Benjamin Baker, and the sources describe it as the first major structure in Britain built of steel. Its three great steel towers, each with long arms reaching out to either side, are the most famous cantilever bridge in the world. This chapter explains what a cantilever is, why balancing two of them works, and what the engineers did to show it to a doubting public.

> **WHAT WAS RUN, AND WHAT TO TRUST**
>
> Every Python snippet in this chapter and in the three exercise solutions was executed, and the output shown in the comments is what Python printed. The bridge's dimensions come from the sources named in the table. The load model is a deliberately simple one: the bridge's average weight spread evenly along its length, with a round 100 m depth for the structure. It shows how the principle works, not what the bridge's designers calculated.

What a Cantilever Is

A **cantilever** is a beam supported at one end only, like a diving board or a balcony. The fixed end must resist both the load's weight and the turning effect it has, which is the whole load times the distance to it. Compare it with the simply supported beam of How Bridges Work 2 using the same span and load:

```
E = 200e3                                   # MPa

def cantilever(w, L):
    """Fixed at one end, uniform load w (kN/m): root moment (kN m) and shear (kN)."""
    return w * L**2 / 2, w * L

def tip_deflection(w, L_m, i_mm4):          # w in kN/m = N/mm
    L = L_m * 1000
    return w * L**4 / (8 * E * i_mm4)

def beam_deflection(w, L_m, i_mm4):
```

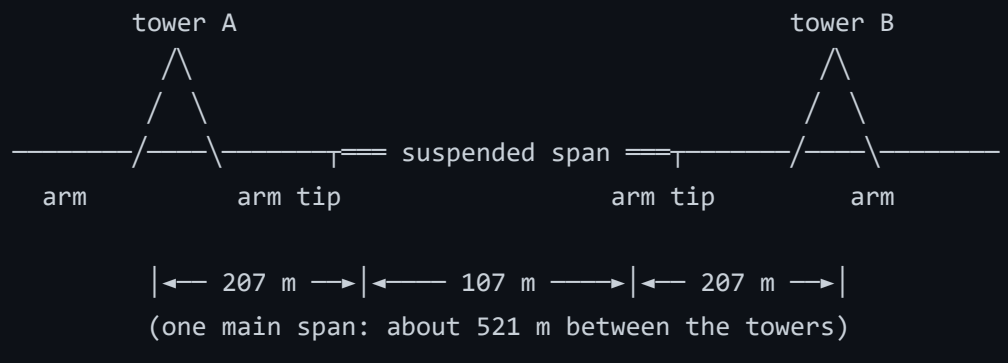
```
    L = L_m * 1000
    return 5 * w * L**4 / (384 * E * i_mm4)

print(cantilever(20, 10))                       # (1000.0, 200)
print(20 * 10**2 / 8, 20 * 10 / 2)              # 250.0 100.0  (simply supported, same
i = 8.0e7
print(round(tip_deflection(20, 10, i)), round(beam_deflection(20, 10, i)))
# 1562 163
```

A 10 m cantilever under 20 kN/m has a root moment of 1000 kN·m and a shear of 200 kN. The simply supported beam of the same length has only 250 kN·m and 100 kN, so the cantilever works **four times as hard** in bending. It also sags far more: 1,562 mm at the tip against 163 mm for the beam, nearly ten times. A lone cantilever is therefore a poor use of material, and the clever idea of the Forth Bridge is to give each cantilever a partner that balances it.

The Balanced Cantilever

Put a tall tower on a pier and send an arm out to each side. If the two arms are equal, their turning effects about the tower cancel, just as two children of equal weight balance on a seesaw. The tower then carries only the two weights, pressing straight down. The arms are in **tension along the top** (they are trying to bend downwards at the tips) and **compression along the bottom**, exactly like the chords of a truss in How Bridges Work 4. The Forth Bridge has three of these "double cantilevers". Between two of them a shorter girder, the **suspended span**, rests on the tips of the arms and carries traffic across the gap.



The numbers for one main span of the Forth Bridge agree between the sources: two cantilever arms of 680 feet (207.3 m) carry a central girder of 350 feet (106.7 m). Add them up: 207.3 + 106.7 + 207.3 = 521.3 m, which matches the reported main span of about 521 m. (The Wikipedia article gives it variously as 1,700 and 1,709 feet; the 1,709 feet figure is the 521 m one.) The table collects the other facts I could check:

FACT	FIGURE	SOURCE NOTE
Opened	4 March 1890; construction from 1882 to December 1889	Wikipedia, Forth Bridge
Designers	Sir John Fowler and Sir Benjamin Baker	Wikipedia
Main spans	two, each about 521 m (arms 207.3 m, suspended span 106.7 m)	Wikipedia; the page gives 1,700 and 1,709 ft in different places
Total length	2,467 m (8,094 ft)	Wikipedia
Height of the towers	110 m (361 ft) above high water	Wikipedia
Rail level	45.7 m (150 ft) above high water	Wikipedia
Rivets	6.5 million	Wikipedia and the Forth Bridges site
Weight	51,324 t of superstructure including rivets (Wikipedia) or 53,000 t of steel (the Forth Bridges site)	the two sources differ; both are quoted
World Heritage	inscribed 5 July 2015	Wikipedia

Working Out the Balance

To get a feel for the forces, take the superstructure's average weight spread along the whole bridge: 51,324 tonnes over 2,467 m is about 204 kN per metre. This is a rough average (the cantilever parts are heavier than the approach spans, so the real figure at the towers is higher), but it is grounded in the bridge's own numbers. The suspended span puts half its weight on each arm tip. The moment at the tower is the arm's own load plus the tip load times the arm length:

```
import math
G = 9.81
w = 51324 * G / 2467                  # average dead load, kN/m (51,324 t over 2,467 m)
a, s = 207.3, 106.7                   # cantilever arm and suspended span, m
print(round(w))                       # 204

p = w * s / 2                         # each arm carries half of the suspended span
m_tower = w * a**2 / 2 + p * a        # hogging moment where an arm meets the tower
reaction = 2 * (w * a + p)            # the tower supports both arms
print(round(m_tower), round(reaction))
# 6642313 106392

span = 2 * a + s                      # one simply supported beam over the same gap
m_beam = w * span**2 / 8
print(round(span, 1), round(m_beam))
# 521.3 6932754

depth = 100                           # m, illustrative (the structure is 110 m tall)
print(round(m_tower / depth))         # force in the top and bottom chord, kN
# 66423
```

The moment at the tower is about 6.64 million kN·m, the tower must carry roughly 106,000 kN (about 10,800 tonnes), and a pair of chords 100 m apart would each carry about 66,000 kN. Look at the middle result: **one beam across the whole 521 m gap would suffer a moment of 6.93 million kN·m, only 4% more.** The cantilever does not reduce the bending much. What it does is change *where* the bending is. In the double cantilever the moment peaks at the tower, where the structure can be made enormously deep (110 m tall), and falls to zero at the tips, where it can be made slim:

```
G = 9.81
w = 51324 * G / 2467
a, s = 207.3, 106.7
p = w * s / 2
m_root = w * a**2 / 2 + p * a

for frac in (0, 0.25, 0.5, 0.75, 1.0):
    x = frac * a                      # distance from the tip of the arm
    m = w * x**2 / 2 + p * x
```

```
    print(frac, round(x, 1), round(m), round(m / m_root, 2))
# 0 0.0 0 0.0
# 0.25 51.8 838354 0.13
# 0.5 103.7 2224857 0.33
# 0.75 155.5 4159510 0.63
# 1.0 207.3 6642313 1.0
```

The columns are the fraction of the way from the tip, the distance from the tip (m), the moment (kN·m) and the moment as a fraction of the value at the tower. Halfway along the arm the moment is only a third of its peak; a quarter of the way in, only 13%. A designer can follow the moment with the depth of the structure: very deep at the tower, tapering to a slender point at the tip. This is the shape of the Forth Bridge, and it is the same logic as the I-beam of How Bridges Work 2, where material goes where the stress is. It also explains the use of steel in two forms: the compression members of the great arms are large tubes, and the tension members are lattice (general knowledge, noted in How Bridges Work 4).

Why a Cantilever, and Why Then

The story of the Forth Bridge begins before Fowler and Baker. In 1871 Thomas Bouch, the engineer whose Tay Bridge later fell, proposed a stiffened steel suspension bridge on roughly the same line. He allowed for a wind pressure of only 10 pounds per square foot on the advice of the Astronomer Royal, the Forth Bridge Railway Act was passed on 5 August 1873, and work began on a brick pier at Inchgarvie in 1878. After the first Tay Bridge collapsed in December 1879, the inquiry found it "badly designed, badly constructed and badly maintained", and Bouch's design for the Forth was formally abandoned on 13 January 1881. The sources I could read do not say in so many words *why* the cantilever was chosen; what they establish is the order of events. Later, the Board of Trade required all bridges to be designed for a wind pressure of 56 pounds per square foot, more than five times Bouch's figure.

One reading, which is my own interpretation and not something the sources state, is that the engineers wanted a design that was visibly **stiff**, with no reliance on a thin cable, and sturdy against wind. A cantilever's deep, braced steel arms and wide base (the structure is 120 ft, about 37 m, wide at the piers) provide exactly that. Another practical gain, which belongs to How Bridges Work 8 as much as this one, is that each arm can be built out from the tower in balance, so that no temporary supports are needed in the water below (general knowledge, not checked in the sources for this chapter).

The Human Cantilever

Fowler and Baker knew that people would doubt a bridge whose arms seemed to hang in the air. In 1887 they gave a famous demonstration, now called the **human cantilever**. The sources describe it this way: the Japanese engineer Kaichi Watanabe sat in the middle, supported between Fowler and Baker, who sat on chairs. Fowler and Baker represent the cantilevers, with their **arms in tension** and the **sticks under compression**, and the **bricks** represent the end piers, weighted with cast iron.

IN THE DEMONSTRATION	IN THE BRIDGE	FORCE
the two engineers on chairs	the two cantilever arms and their towers	carry the weight
their outstretched arms	the top members of the arms	tension
the sticks pressing against the chairs	the compression members below	compression
Watanabe in the middle	the suspended span	the load
the bricks, weighted with cast iron	the weighted ends of the anchor arms	stop the whole thing tipping over

It is a brilliant piece of teaching: anyone can feel their own arms pulling and the sticks pushing, and see that nothing needs to hold the middle man except the two arms and the bricks behind them. The counterweights at the ends matter as much as the arms, which Exercise 3 explores with numbers. (I describe the demonstration only as far as the source states it; the text does not give more detail than this.)

> Common mistakes
> Treating a cantilever as a beam with a small moment (it is four times larger than the simply supported beam of the same length); forgetting the anchor arm or counterweight, which stops the arm from tipping the structure over; and treating the Forth Bridge's average weight as the real weight along each arm (the arms are heavier than the approaches). A note on human cost: records of the deaths during construction vary, and a summary of the Wikipedia article gives 73 as of 2009, a figure I could not verify directly, so treat it as unconfirmed.

Where This Leads

The cantilever passes load through deep, rigid steel arms. The next chapter looks at the opposite approach, the one that Bouch proposed for the Forth: hang the deck from flexible cables. It follows the catenary curve of a cable, the anchorages that hold it, the towers that lift it, and the Forth Road Bridge of 1964, which stands next to its older neighbour.

EXERCISE 1

A steel cantilever arm projects 6 m. It carries 12 kN/m and a 20 kN point load at its tip; $I = 1.5 \times 10^8$ mm^4 and $E = 200{,}000$ MPa. Find the moment and shear at the root, and the tip deflection (uniform load: $wL^4/(8EI)$; tip load: $PL^3/(3EI)$). Compare it with an assumed limit of span/180 and say what I would be needed to pass.

View solution

EXERCISE 2

Using the chapter's average load of about 204 kN/m and an arm of 207.3 m on each side of a tower, calculate the moment at the tower, the tower's reaction and the chord force for a 100 m depth, for suspended spans of 0, 53, 106.7 and 160 m resting on the arm tips. How much does each extra 50 m of suspended span add to the moment?

View solution

EXERCISE 3

A 100 m arm carries 80 kN/m and a 2000 kN tip load. On the other side of the pier an anchor arm of length b carries only its own 80 kN/m. For b = 60, 80, 100 and 120 m, find the overturning moment about the pier, the factor against tipping from the anchor arm alone, and the counterweight at its end that gives a factor of 1.5. How long would the anchor arm have to be to balance the main arm with no counterweight?

View solution

QUICK REFERENCE

- **Cantilever:** fixed at one end; moment $wL^2/2$ (four times a simply supported beam of the same length and load)
- **Balanced cantilever:** equal arms on each side of a tower, so the turning effects cancel
- **Forth Bridge:** opened 4 March 1890; Fowler and Baker; three double cantilevers; two main spans of about 521 m
- **One main span:** arm 207.3 m + suspended span 106.7 m + arm 207.3 m
- **Moment:** peaks at the tower and falls to zero at the tip, so the arms taper
- **Human cantilever (1887):** arms in tension, sticks in compression, weighted bricks as the end piers
- **Anchor arm or counterweight:** stops the cantilever tipping

COMING NEXT

How Bridges Work 7 is about suspension bridges: a cable's curve (the catenary), the anchorages and towers, deck stiffness, and the Forth Road Bridge of 1964.

How Bridges Work

Chapter 7 · Suspension Bridges

The Forth Bridge of 1890 carries its load through deep steel arms. A suspension bridge does the opposite: it hangs the road from flexible steel cables that carry the load in pure **tension**, with no bending in the cables at all. How Bridges Work 5 showed that steel is the strongest material in tension per unit of weight, and a cable is the shape that makes the most of it. This chapter follows the load from the deck up the hangers into the main cable, over the towers and down into the anchorages, using the Forth Road Bridge of 1964, standing just beside its older neighbour, as the running example.

> **WHAT WAS RUN, AND WHAT TO TRUST**
>
> Every Python snippet in this chapter and in the three exercise solutions was executed, and the output shown in the comments is what Python printed. The Forth Road Bridge figures in the table come from the source named; the *loads* and the *sag* used in the calculations are illustrative assumptions, labelled where they appear. The results show how the principle works, not the designers' figures.

The Load Path of a Suspension Bridge

```
deck (the road)
   │
   ▼  hangers (tension)
main cable            ← tension, along a smooth curve
   │
   ▼  over the tops of the towers
towers                ← compression, straight down
   │
   ▼
foundations

main cable continues past the towers
   │
   ▼
anchorages            ← resist the cable's pull (weight and rock)
```

Compare this with How Bridges Work 1's load path for a beam. Here the deck does not carry its load to the ends. It hangs it on the cable every few metres, and the cable carries it to the towers and the anchorages. The towers are in compression, the cables and hangers in tension, and the anchorages hold the whole system from pulling in. The span can be very long because the cable is a small cross-section of steel working in the way steel does best.

The Forth Road Bridge: The Facts

FACT	FIGURE
Opened	4 September 1964, by Queen Elizabeth II and the Duke of Edinburgh; the construction plan was accepted in February 1958 and work began that September
Design	Mott, Hay and Anderson with Freeman Fox & Partners
Builders	a joint venture of Sir William Arrol & Co., Cleveland Bridge & Engineering Company and Dorman Long
Cost	£15.1 million
Main span	1,006 m (3,301 ft)
Side spans	408 m each
Total length	2,512 m (8,241 ft)
Towers	156 m (512 ft) above mean water level
Main cables	590 mm in diameter, each with 11,618 high-tensile wires of 5 mm; the page says each cable carries 13,800 tonnes of the bridge's load
Materials	39,000 tonnes of steel; the page gives two conflicting concrete figures (115,000 m^3 and 210,000 tons)
Construction deaths	seven people died

(Source: the Wikipedia article on the Forth Road Bridge. A subagent read the first 39,400 of 97,257 characters verbatim, plus a model summary of the rest, and the figures above are all from the verbatim part. The page does not state the sag of the cables, the clearance above the water, the anchorage method or the deck type, so this chapter does not guess them.) Note the main

span: at 1,006 m it is almost twice the 521 m main span of the Forth Bridge (arms plus girder, How Bridges Work 6), and it crosses the same estuary beside it.

The Shape of a Cable

A cable has no stiffness of its own, so it can only take the shape in which its tension balances the loads. For the loads on a suspension bridge there are two shapes to know. A cable hanging under only its **own weight** forms a **catenary**. A cable carrying a **deck** hung from it at regular intervals, so that the load is spread evenly along the *horizontal* span, forms a **parabola**. On a real bridge the deck weighs far more than the cable, so the parabola is the right model for the finished bridge (the catenary matters during construction, before the deck is hung). The two curves are very close, as the code further below shows.

For a parabolic cable of span `L` and sag `f` under a uniform load `w` per metre, the horizontal pull is `H = wL²/(8f)`. It is the same formula as the arch thrust of How Bridges Work 3, with the sag in place of the rise: a cable is an upside-down arch. The vertical pull at each tower is `V = wL/2`, and the biggest tension, at the tower, is the combination of the two: `T = √(H² + V²)`.

```
import math

def cable(w, span, sag):
    """Parabolic cable under a uniform load w (kN/m) along the span (m)."""
    h = w * span**2 / (8 * sag)                    # horizontal pull, kN
    v = w * span / 2                               # vertical pull at each tower, kN
    t = math.hypot(h, v)                           # largest tension, at the tower
    length = span * (1 + 8 / 3 * (sag / span) ** 2)   # approximate cable length, m
    return h, v, t, length

span = 1006
sag = span / 11                                    # assumed sag-to-span ratio of 1/1
h, v, t, length = cable(90, span, sag)
print(round(sag, 1), round(h), round(v), round(t), round(length))
# 91.5 124492 45270 132468 1028
wires = 11618 * math.pi * 2.5**2                   # steel area in one cable, mm2
print(round(wires), round(t * 1000 / wires))       # area, and the stress in MPa
# 228119 581
```

This models one main cable over the 1,006 m span with two *assumptions*: a sag of 1/11 of the span (the sources I found put typical suspension bridge sags somewhere between about 1/8 and 1/15 of the span, and they do not agree exactly) and a load of 90 kN per metre per cable, chosen so that the answer lands near the 13,800 tonnes the article quotes. The result: a horizontal pull of 124,492 kN, a vertical pull of 45,270 kN at each tower, and a largest tension of 132,468 kN, which is about 13,500 tonnes-force. The 11,618 wires of 5 mm have a total steel area of 228,119 mm², so the stress in the wire is 581 MPa. That is a sensible fraction of what high-tensile bridge wire can take (general knowledge: its breaking strength is of the order of 1,500 MPa or more), consistent with a factor of safety of two or more. The cable is 1,028 m long, only 22 m longer than the straight span.

Flatter Cables Pull Harder

The sag is a design choice, and it controls everything. Because `H = wL²/(8f)`, a flatter cable pulls harder. The same bridge with different sags:

```
for ratio in (6, 8, 11, 15, 25):
    h, v, t, length = cable(90, span, span / ratio)
    print(f"1/{ratio:<3}", round(h), round(t), round(length))
# 1/6   67905 81612 1081
# 1/8   90540 101227 1048
# 1/11  124492 132468 1028
# 1/15  169762 175695 1018
# 1/25  282938 286536 1010
```

The columns are the sag ratio, the horizontal pull (kN), the largest tension (kN) and the cable length (m). Going from 1/6 to 1/25 more than quadruples the horizontal pull (67,905 to 282,938 kN) and more than triples the cable tension, while shortening the cable by 71 m. A deeper sag means a lighter cable but a **taller tower** to hold it up; a flatter sag means a heavier cable and bigger anchorages but shorter towers. Choosing the sag is a trade-off between tower cost and cable cost, the same trade-off as the rise of an arch.

Catenary or Parabola?

How different are the two curves for this bridge? The code finds the catenary that has the same sag, by repeatedly halving an interval until the match is close (bisection), and compares the two curves a quarter of the way across from a tower, measured as the drop below the tower top:

```
def catenary_sag(a, half):
    return a * (math.cosh(half / a) - 1)

def solve_a(span, sag):
    lo, hi = 1.0, 1e7
    for _ in range(200):                  # bisection: find a so that the sag matches
        mid = (lo + hi) / 2
        if catenary_sag(mid, span / 2) > sag:
            lo = mid
        else:
            hi = mid
    return (lo + hi) / 2

sag = span / 11
a = solve_a(span, sag)
x = span / 4                              # a quarter of the way across
y_cat = catenary_sag(a, span / 2) - catenary_sag(a, span / 2 - x)
y_par = sag * (1 - (1 - x / (span / 2)) ** 2)   # same-sag parabola
print(round(a), round(y_cat, 2), round(y_par, 2), round(y_cat - y_par, 2))
# 1398 68.77 68.59 0.18
```

The catenary's parameter is 1,398 m. A quarter of the way across, it hangs 68.77 m below the tower top, and the parabola 68.59 m: the two curves differ by **18 cm** over a span of a kilometre. That is why the parabola is the standard model for a cable with a deck. The catenary still matters for the cable before the deck is hung, and for cables with a large sag-to-span ratio, where the gap grows (Exercise 3).

Towers and Anchorages

The cable passes over the tops of the **towers**, which carry the vertical pull of both sides straight down in compression (here 45,270 kN from each side). On a suspension bridge the towers are therefore slender columns in compression, and need to be stiff against buckling (How Bridges Work 4). At the far ends the cable must be fixed. The **anchorage** has to resist the full horizontal

pull `H` that the cable exerts. Many anchorages do this with sheer weight and friction, and others are set into rock. A rough estimate of the weight of a gravity anchorage that resists sliding by friction alone, for the cable pull above, looks like this:

```
def anchorage(h_kn, mu=0.6, factor=1.5, rho=2400):
    weight_kn = factor * h_kn / mu                    # needed weight, kN
    mass_t = weight_kn / 9.81
    return round(mass_t), round(mass_t * 1000 / rho)  # tonnes, m3 of concrete

print(anchorage(124492))
# (31726, 13219)
```

About 31,700 tonnes of concrete, or roughly 13,200 m^3 per anchorage, with an assumed friction coefficient of 0.6 and a factor of safety of 1.5. This model is very crude: it ignores the vertical component of the pull and the way a real anchorage is shaped and founded. The sources do not describe the Forth Road Bridge's anchorage, so this is a general illustration only. The lesson is the size of the numbers: the horizontal pull of the cables has to be matched by something enormous.

The Deck: Why Stiffness Matters

A cable hung with a perfectly even load keeps its parabolic shape. A real load is not even: a heavy lorry in one place pulls the cable down there and lets it rise elsewhere, and wind pushes the whole deck sideways. A cable alone would swing and sag in response. A **stiffening girder or truss** along the deck spreads a local load over many hangers, so that the cable feels something near an even load and the deck stays flat. Bouch's 1871 proposal for the Forth, mentioned in How Bridges Work 6, was a *stiffened* suspension bridge. The stiffness of the deck also controls how it moves in the wind, which How Bridges Work 9 takes up with the Tacoma Narrows bridge. The Forth Road Bridge article I read does not state the deck type of the bridge (truss or box girder), so I do not describe it here.

> Common mistakes
> Using the catenary for a cable with a deck (the right model is a parabola); using the tower's vertical reaction as the cable tension (the tension at the tower is the combination of horizontal and vertical pulls); forgetting that the horizontal pull falls as the sag increases (flatter means more pull); and

trusting a number from a source without asking what it measures (the article's "13,800 tonnes" per cable is described only as the load each cable carries).

Where This Leads

A bridge's cables are out of sight and hard to inspect, so the story has a sequel: an inspection programme that began in 2003 led to the discovery in 2005 of potentially serious problems in the Forth Road Bridge's main cables, and a later study reported an 8% to 10% loss of strength, and a dehumidification system was installed to keep the air among the strands below 40% humidity. That story, and what it taught, belongs to How Bridges Work 9. The next chapter returns to the third Forth bridge: a design that keeps the towers and the tension but drops the draped main cable and runs straight cables from the towers to the deck.

EXERCISE 1

A small suspension bridge has a main span of 300 m and two cables, each carrying 60 kN/m (deck and cable together). For sag-to-span ratios of 1/8, 1/10 and 1/12, find the horizontal pull, the vertical pull at each tower, the largest tension, the approximate cable length, and the steel area needed at a working stress of 600 MPa. Which sag needs the least steel, and what do you pay for that choice?

View solution

EXERCISE 2

Each cable of the bridge in Exercise 1 ends in a gravity anchorage that must resist the horizontal pull by friction alone (friction coefficient 0.6, factor of safety 1.5, concrete of 2400 kg/m^3). For the three sag ratios find the weight, mass and volume of concrete, then repeat for friction coefficients of 0.4 and 0.8 at the 1/10 sag. How does the anchorage size depend on the pull and on the friction?

View solution

EXERCISE 3

For a 300 m span with sag-to-span ratios of 1/20, 1/10 and 1/5, find the catenary parameter that gives each sag (use bisection), and the difference in metres between the catenary and the same-sag parabola a quarter of the way across. For which ratio does the choice of curve matter most?

View solution

QUICK REFERENCE

- **Load path:** deck, hangers (tension), main cable (tension), towers (compression), anchorages
- **Cable with a deck:** a parabola; **cable under its own weight:** a catenary
- **Pulls:** $H = wL^2/(8f)$, $V = wL/2$ at each tower, $T = \sqrt{(H^2 + V^2)}$
- **Sag:** flatter means more pull and more cable steel; deeper means taller towers
- **Anchorage:** must resist H; weight, friction and rock
- **Forth Road Bridge (1964):** main span 1,006 m, towers 156 m, cables of 11,618 wires of 5 mm
- **Stiffening girder:** spreads local loads and steadies the deck against wind

COMING NEXT

How Bridges Work 8 is about cable-stayed bridges: fan and harp layouts, the Queensferry Crossing of 2017, construction by balanced cantilever, and wind shielding.

How Bridges Work

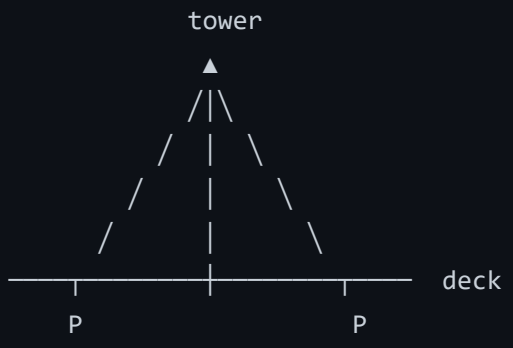
Chapter 8 · Cable-Stayed Bridges

A suspension bridge hangs the deck from a draped main cable. A **cable-stayed bridge** skips the drape: each **stay** is a straight steel cable that runs from a tower directly to a point on the deck. The towers are still in compression and the stays in tension, but now every stay is its own short support, so there is no huge main cable and no giant anchorage at the ends. The third Forth bridge, the Queensferry Crossing, opened in 2017, is the one this chapter follows. It also brings the course's main theme to a close: the same steel, forces and mathematics, arranged in a third way across the same water.

> **WHAT WAS RUN, AND WHAT TO TRUST**
>
> Every Python snippet in this chapter and in the three exercise solutions was executed, and the output shown in the comments is what Python printed. The facts about the Queensferry Crossing come from the sources named. The **loads, tower height above the deck, number of stays and segment weights in the calculations are illustrative assumptions**, labelled where they appear, so the numbers show how the principle works, not the designers' figures.

How a Stay Works

A stay holds up one point of the deck. It is tilted at an angle `θ` above the deck, so its tension `T` has two parts: a vertical part that lifts the deck (`T sin θ = P`, the load it carries) and a horizontal part that pulls the deck towards the tower (`T cos θ = P / tan θ`). The horizontal parts, added up along the deck, put the whole deck into **compression**, like a long strut. The tower receives the vertical parts and carries them down in compression.




```
each stay: tension T, with the load P taken by its vertical part
```

```
    and a horizontal push on the deck that adds up along it
```

Fan and Harp

Two arrangements are the classic pair. In a **fan** the stays all converge on one point at the top of the tower. In a **harp** the stays are parallel to one another, like the strings of a harp, each meeting the tower at a different height. A **semi-fan** is a compromise: the stays are spread down the upper part of the tower. The sources I found say that the fan gives the smaller moment on the tower but puts a large concentration of force where the stays meet it, and that the semi-fan is often preferred when there are many cables. The code below compares the first two in a simple model of one side of a tower, with the deck load shared equally by ten stays. The tower height above the deck, the stay spacing and the load are illustrative assumptions:

```
import math

def stays(layout, n, d, height, p):
    """Forces in the n stays on one side of a tower. Stay i holds a deck point
    i*d metres from the tower, and carries a vertical load p (kN)."""
    out = []
    for i in range(1, n + 1):
        x = i * d
        if layout == "fan":
            y = height                          # every stay meets the tower top
        else:
            y = height * x / (n * d)            # harp: parallel stays
        length = math.hypot(x, y)
        t = p * length / y                      # T sin(theta) = p
        out.append((x, y, length, t, t * x / length))   # last item: horizontal part
    return out

n, d, height, p = 10, 32.5, 150, 8000           # illustrative
for layout in ("fan", "harp"):
    rows = stays(layout, n, d, height, p)
    h_sum = sum(r[4] for r in rows)             # total push along the deck, kN
    steel = sum(r[3] * r[2] for r in rows) / 600e3   # m3, at 600 MPa
    print(layout, round(max(r[3] for r in rows)), round(h_sum), round(steel, 1))
# fan 19090 95333 56.1
```

```
# harp 19090 173333 62.6
```

The columns are the layout, the largest stay force (kN), the total horizontal push the stays put on the deck (kN), and the steel volume of all ten stays (m^3) at an assumed 600 MPa. In this model the two layouts have the *same* largest stay force (the outermost stay is the same in both), but the harp pushes the deck **82% harder** (173,333 against 95,333 kN) and needs about 12% more steel. The reason is geometry: a fan has many steep stays near the tower, which are efficient, whereas every harp stay has the same shallow slope. Real bridges choose layouts for many reasons (looks, the room on the tower to connect to, how the loads change along the span), so this is a simple comparison, not a verdict.

The Best Stay Angle

Why not make every stay as steep as possible? A steep stay needs a taller tower. A shallow stay is cheaper in tower but pulls harder and pushes the deck harder. For a stay holding a deck point at a fixed horizontal distance, there is a best angle, where the stay's steel volume (tension × length ÷ stress) is smallest:

```
def stay_volume(theta_deg, p=8000, x=100, stress=600e3):
    """Steel volume (m3) of one stay holding load p at horizontal distance x."""
    th = math.radians(theta_deg)
    t = p / math.sin(th)                  # tension, kN
    length = x / math.cos(th)             # metres
    return t * length / stress

for deg in (20, 30, 40, 45, 50, 60, 70):
    th = math.radians(deg)
    print(deg, round(8000 / math.sin(th)), round(8000 / math.tan(th)), round(stay_volume
# 20 23390 21980 4.15
# 30 16000 13856 3.08
# 40 12446 9534 2.71
# 45 11314 8000 2.67
# 50 10443 6713 2.71
# 60 9238 4619 3.08
# 70 8513 2912 4.15
```

The columns are the angle (degrees), the tension (kN), the horizontal push on the deck (kN) and the steel volume (m^3). The volume is lowest at **45°** (2.67 m^3) and rises either side: at 20° the stay needs 4.15 m^3 and pushes the deck with 21,980 kN, nearly three times the push at 45°. This is a useful guide, because it explains why stays on real bridges tend to be neither very flat nor very steep. It ignores the cost of the tower and of the deck's compression, so designers adjust it.

The Queensferry Crossing

FACT	FIGURE
Opened	to traffic on 30 August 2017; formally opened by Queen Elizabeth II on 4 September 2017, 53 years after she opened the Forth Road Bridge
Construction	preparatory works began in September 2011; finished 27 August 2017
Design	Ramboll leading a design joint venture with Sweco and Leonhardt, Andrä and Partners; Naeem Hussain of Arup was the lead design engineer
Contractor	Forth Crossing Bridge Constructors (Dragados, Hochtief, American Bridge, Morrison Construction)
Type	a three-tower cable-stayed bridge; at opening, the longest triple-tower cable-stayed bridge in the world
Towers	207 m (679 ft) high each
Longest span	650 m (2,133 ft)
Total length	2,700 m (8,858 ft), including approaches
Deck	149 segments, each 12 m long, built in China and Spain and shipped in October 2013 (the page also gives the width as 40 m, which looks wrong for a road deck)
Cost	£1.35 billion (infobox)
Traffic	motorcycles, cars and heavy goods vehicles; public transport, cyclists and pedestrians use the Forth Road Bridge

(Source: the Wikipedia article on the Queensferry Crossing, read in full by a subagent apart from its reference list. The same page does *not* state the number of spans or stays, the stay layout, the deck type, the construction method or the foundations.) For those, I used a separate search, with

the reliability noted: **the stays cross at midspan** (Hochtief's project page describes a zone of crossed cables at mid-span that gives extra strength and stiffness, allowing a lighter deck and slimmer towers, with strands replaceable one at a time); one source gives **288 stays** in total; and the deck was built by **balanced cantilever**. I read search summaries, not the full pages, so treat these items as less certain than the table.

Building by Balanced Cantilever

How Bridges Work 6's balanced cantilever idea appears again here, for a different reason. The deck of a cable-stayed bridge is built out from each tower, one segment at a time, with each new segment hung from a new stay as soon as it is in place. A search turned up descriptions of segments being lifted from barges by equipment on the towers and added **at alternate ends**, so that the cantilever stays balanced and the tower is not overloaded, and a report that the pair of freestanding cantilevers at the central tower reached about 644 m tip to tip, which was recognised as a world record. That agrees with the 650 m longest span. The key calculation is how much out-of-balance load the tower can stand:

```
seg_len, seg_weight = 12, 3000          # m and kN (illustrative: 250 kN/m x 12 m)

def unbalance(arm_m, segments_ahead=1):
    """Moment on the tower (kN m) when one side is ahead by whole segments."""
    return segments_ahead * seg_weight * arm_m

for arm in (96, 192, 288, 324):
    print(arm, unbalance(arm), unbalance(arm, 2))
# 96 288000 576000
# 192 576000 1152000
# 288 864000 1728000
# 324 972000 1944000
```

The columns are the arm length (m), the moment on the tower if one side is one segment ahead (kN·m) and two segments ahead. The moment is proportional to the arm: one segment ahead at 324 m (972,000 kN·m) puts **3.4 times the load on the tower** of the same segment at 96 m (288,000). This is why the segments are added in turn from alternate ends, and why the whole job is sensitive to the weather. The same search source gave a wind limit of **11 m/s** (about 25

mph) while the stays were being installed, and the Wikipedia article reports 25 days lost to high winds in April and May 2016 alone.

Wind Shielding

The Forth Road Bridge was often restricted in high winds, and the Wikipedia article says that wind shielding "has been built into the design" of the Queensferry Crossing so that it can be used in high winds. The sources I found do not agree on the details: one says 3 m high louvred shields run the full length of the deck, another describes 3.6 m high sides; one says cars can cross in winds up to 100 mph where the old bridge's restriction was 80 mph, and another says 115 mph for all vehicles. What they all rely on is simple physics: wind pressure grows with the **square** of the speed.

```
rho = 1.225                          # air, kg/m3
MPH = 0.44704                        # metres per second per mph

def pressure_kpa(mph):
    v = mph * MPH
    return 0.5 * rho * v**2 / 1000

for mph in (50, 80, 100, 115):
    print(mph, round(mph * MPH, 1), round(pressure_kpa(mph), 2))
# 50 22.4 0.31
# 80 35.8 0.78
# 100 44.7 1.22
# 115 51.4 1.62
print(round(pressure_kpa(100) / pressure_kpa(80), 2))
# 1.56
```

The columns are the speed in mph, the speed in m/s and the dynamic pressure (kPa) of the wind. At 100 mph the pressure is 1.22 kPa against 0.78 kPa at 80 mph: a 25% increase in speed gives a **56% increase in force**. A solid barrier cuts the wind the vehicles feel, but it also takes some of that pressure itself, and the deck and towers must be designed for it. The same Wikipedia article records a different kind of weather problem: on 11 February 2020 the bridge was closed for the first time since it opened, when ice fell from the towers and damaged eight vehicles.

Three Bridges, Three Ideas

BRIDGE	OPENED	TYPE	LONGEST SPAN	HOW THE LOAD TRAVELS
Forth Bridge	1890	cantilever	about 521 m	deep steel arms in tension (top) and compression (below), balanced over each tower
Forth Road Bridge	1964	suspension	1,006 m	deck hangs from a draped main cable, which runs to towers and anchorages
Queensferry Crossing	2017	cable-stayed	650 m	straight stays from three towers to the deck, which is in compression

Common mistakes

Forgetting that the stays push the deck in compression (the deck is not just carried: it is also a strut); using the stay's vertical load as its tension (the tension is larger by 1 / sin θ); treating a model result as the designers' figure (the numbers here are illustrations); and relying on one source for a detail (the stay count and the wind speeds in this chapter each come from one or two web summaries).

Where This Leads

You now have all three Forth bridges in the toolkit. The chapters so far have asked whether a bridge is strong enough for the loads we expect. The next chapter asks what happens when it meets loads we did not expect or understand: wind that sets a deck swinging, water that undermines a pier, and cables that corrode out of sight. It looks at the Tacoma Narrows bridge, the myths about resonance, scour, and what the Forth Road Bridge's cable story taught engineers.

EXERCISE 1

A tower 150 m above the deck carries a half-span of 325 m, with a total deck load of 80,000 kN on that side shared equally by *n* evenly spaced stays. For n = 6, 10 and 14, compare the fan and the harp: the largest stay force, the total horizontal push on the deck and the steel volume of all the stays at 600 MPa. What happens to the push in the harp as n changes, and why?

View solution

EXERCISE 2

A stay holds a vertical load of 5000 kN at a deck point 150 m from the tower base. For angles from 20° to 70° in 5° steps, find the tension, the horizontal push on the deck and the steel volume of the stay (tension × length ÷ 600 MPa). Find the angle with the smallest volume, and say what a shallower stay costs.

View solution

EXERCISE 3

Each deck segment weighs 3000 kN, and the tower can take an out-of-balance moment of 700,000 kN·m (both illustrative). If one side gets *k* segments ahead, the moment is k × 3000 × arm length. For arms of 96, 192, 288 and 324 m, find the largest whole number of segments one side may be ahead and the moment it causes. What does the result say about building long cantilevers?

View solution

QUICK REFERENCE

- **Stay:** a straight cable from tower to deck; tension T = P / sin θ; horizontal part P / tan θ
- **Deck:** pushed into compression by the stays; the tower carries their vertical parts
- **Fan:** stays meet at the tower top; **harp:** parallel stays; **semi-fan:** a compromise
- **Stay angle:** steel volume is lowest near 45° (a simple guide)
- **Queensferry Crossing (2017):** three towers of 207 m, longest span 650 m, total length 2,700 m
- **Balanced cantilever construction:** segments added from alternate ends; out-of-balance moment grows with arm length
- **Wind pressure:** 0.5 × air density × speed^2; a 25% rise in speed is a 56% rise in pressure

COMING NEXT

How Bridges Work 9 is about wind, water and failure: the Tacoma Narrows bridge, the myths about resonance, scour, fatigue, the Forth Road Bridge's cable corrosion findings, and what engineers learned from failures.

How Bridges Work

Chapter 9 · Wind, Water and Failure

Almost every famous bridge failure has the same shape: the loads the engineers checked were fine, and the one that destroyed the bridge was not on the list. The first Tay Bridge fell in a gale, and its inquiry found it badly designed, badly constructed and badly maintained. The Tacoma Narrows bridge twisted itself apart in a wind that was not a storm. Rivers carry away the ground that foundations stand on. And cables corrode out of sight. This chapter takes the four in turn, with a few small simulations that show why they are dangerous, and ends with what each failure added to the engineer's checklist.

WHAT WAS RUN, AND WHAT TO TRUST

Every Python snippet in this chapter and in the three exercise solutions was executed, and the output shown in the comments is what Python printed. Facts about the bridges come from the sources named. The models are **deliberately simple illustrations with assumed numbers** (damping, soil strength, wire strength), so they show how the effects work, not what happened at any particular bridge.

Tacoma Narrows, 1940

FACT	WHAT THE SOURCES SAY
Dates	opened 1 July 1940; collapsed 7 November 1940
Main span	2,800 ft (about 853 m)
Width	"just 39 feet (12 m) wide", two lanes
Deck stiffening	Moisseiff proposed 8 ft (2.4 m) deep plate girders instead of the 25 ft (7.6 m) deep trusses the state had proposed
Wind at collapse	about 40 mph (64 km/h); flutter occurred for any sustained wind above about 35 mph (56 km/h)
Cause	"moderate winds produced aeroelastic flutter that was self-exciting and unbounded", a twisting mode in which the two halves of the deck twisted in opposite directions
Casualties	no human deaths are stated; a dog, Tubby, died and people were injured

FACT	WHAT THE SOURCES SAY
Afterwards	a new bridge opened in 1950 using the original towers' pedestals and cable anchorages; the page says the collapse boosted research into bridge aerodynamics, which has influenced the designs of all later long-span bridges

(Source: the Wikipedia article on the 1940 Tacoma Narrows Bridge. A subagent read the first 39,400 of 95,062 characters verbatim, and these facts are from that part. The article's own introduction says construction began in September 1938 while its infobox says 23 November 1938, so I give no start date. The footage of the collapse, shot by two cameramen and added to the US National Film Registry in 1998, is often shown about 50% too fast, because of a frame-rate conversion error, which makes the motion look more violent than it was.)

Look at the shape. The deck was 39 ft wide and only 8 ft deep over a span of 2,800 ft, a slender strip, much more flexible than the 25 ft trusses first proposed. The code gives the proportions:

```
span, width = 2800, 39                    # ft: Tacoma Narrows main span and deck width
for name, depth in (("8 ft plate girders", 8), ("25 ft trusses", 25)):
    print(name, "depth/span = 1/%d" % round(span / depth), " width/span = 1/%d" % round(
# 8 ft plate girders depth/span = 1/350  width/span = 1/72
# 25 ft trusses depth/span = 1/112  width/span = 1/72
```

The built deck had a depth of 1/350 of the span, against 1/112 for the trusses. A shallow deck is a flexible deck (the stiffness of a beam depends on the cube of its depth, How Bridges Work 2), and a narrow one twists easily. Both properties made the bridge responsive to the wind.

Resonance or Flutter?

Many physics textbooks use the Tacoma Narrows collapse to illustrate **resonance**: a regular push at just the right rhythm makes a swing go higher and higher. The Wikipedia article says the event "is presented as an example of elementary forced mechanical resonance, but it was more complicated in reality". The cause was **aeroelastic flutter**, and the difference matters:

	RESONANCE	FLUTTER
The push	an outside force with its own rhythm	the wind's force depends on the structure's own motion
Growth	levels off, limited by damping	keeps growing if the wind feeds in more energy than damping removes
Needs	the right frequency	a steady wind above a critical speed

A small simulation shows the difference. The structure is an oscillator with a natural frequency of 0.2 Hz and a damping ratio of 1% (both assumed). First it is pushed by a rhythmic force at its own frequency (resonance). Then the push is removed and the wind instead supplies "negative damping", an assumed energy input that is bigger than the structure's own damping, so that the net damping is below zero. The code steps through time and reports the biggest swing in each 50 s window:

```
import math

def run(zeta, omega, a, force, omega_f, t_end, dt=0.01):
    """x'' + (2 zeta omega - a) x' + omega^2 x = force cos(omega_f t), by RK4.
    a is the 'negative damping' fed in by the wind. Returns (time, x) samples."""
    c = 2 * zeta * omega - a
    def f(t, x, v):
        return v, force * math.cos(omega_f * t) - c * v - omega**2 * x
    t, x, v, out = 0.0, 0.0, 0.001, []
    while t < t_end:
        k1x, k1v = f(t, x, v)
        k2x, k2v = f(t + dt/2, x + dt/2*k1x, v + dt/2*k1v)
        k3x, k3v = f(t + dt/2, x + dt/2*k2x, v + dt/2*k2v)
        k4x, k4v = f(t + dt, x + dt*k3x, v + dt*k3v)
        x += dt/6 * (k1x + 2*k2x + 2*k3x + k4x)
        v += dt/6 * (k1v + 2*k2v + 2*k3v + k4v)
        t += dt
        out.append((t, x))
    return out

def peak(samples, t0, t1):
    return max(abs(x) for t, x in samples if t0 <= t < t1)

omega, zeta = 2 * math.pi * 0.2, 0.01
forced = run(zeta, omega, 0.0, 0.1, omega, 400)            # resonance: periodic push
```

```
print([round(peak(forced, t, t + 50), 2) for t in (0, 100, 200, 300)])
# [1.45, 2.68, 3.03, 3.13]
print(round(0.1 / (2 * zeta * omega**2), 2))               # theory: F / (2 zeta omega^
# 3.17

flutter = run(zeta, omega, 0.06, 0.0, omega, 400)          # wind feeds energy in
print([round(peak(flutter, t, t + 50), 3) for t in (0, 100, 200, 300)])
# [0.002, 0.011, 0.061, 0.348]

calm = run(zeta, omega, 0.0, 0.0, omega, 400)              # same, with no wind effect
print([round(peak(calm, t, t + 50), 4) for t in (0, 100, 200, 300)])
# [0.0008, 0.0002, 0.0001, 0.0]
```

Read the four lines. **Resonance** grows fast at first but flattens out: the swing approaches 3.13 m, close to the theoretical limit of 3.17 m, because the structure's damping removes energy as fast as the push supplies it. **Flutter** starts from a tiny disturbance of 0.002 m and grows by a factor of about 5.7 every 100 s (it doubles every 40 s or so), reaching 0.348 m after 300 s *and still accelerating*: nothing in the model limits it, as the article's phrase "self-exciting and unbounded" says. The last line shows the same bridge without the wind effect: any disturbance simply dies away. The model is linear and ignores the limits that real structures set, so it is an illustration of the mechanism, not of Tacoma's actual motion. Exercise 1 explores how a critical wind speed arises.

Water: Scour

A river bed is not fixed. **Scour** is the removal of sand and gravel from around the foundations of piers and abutments by fast-moving water. The Wikipedia article calls it one of the three main causes of bridge failure (the others being collision and overloading), and gives these estimates: 60% of all bridge failures result from scour and other hydraulic causes (an estimate from a 1992 conference paper), and in the US 46 of 86 major bridge failures between 1961 and 1976 resulted from scour near piers. Scour can be at its worst during a flood, which is also when it is hardest to see. The great flood of 1829 that took the parapets off the Carrbridge packhorse bridge in How Bridges Work 3 is the kind of event in which a river does this kind of work.

The effect on a foundation is easy to model. A friction pile stands on the grip of the soil along its length, and scour removes the upper part of that length, so its capacity falls in proportion. The

code assumes a 1.0 m diameter pile, 12 m long, a skin friction of 60 kPa and a working load of 1,000 kN (all illustrative):

```
import math

def pile_factor(length_m, scour_m, load_kn, dia_m=1.0, skin_kpa=60):
    """Safety factor of a friction pile after the river bed drops by scour_m."""
    embedded = max(0.0, length_m - scour_m)
    capacity = skin_kpa * math.pi * dia_m * embedded      # kN
    return capacity / load_kn

for scour in (0, 2, 4, 6, 8):
    print(scour, round(pile_factor(12, scour, 1000), 2))
# 0 2.26
# 2 1.88
# 4 1.51
# 6 1.13
# 8 0.75
# scour depth at which the factor falls to 1.5 and to 1.0
for target in (1.5, 1.0):
    d = 12 - target * 1000 / (60 * math.pi * 1.0)
    print(target, round(d, 2))
# 1.5 4.04
# 1.0 6.69
```

The safety factor falls from a healthy 2.26 to 1.51 after 4 m of scour and to 1.13 after 6 m, and the pile fails at 6.69 m, more than half its length. Nothing visible happens above the water until it is too late. The article lists many countermeasures. **Riprap** (large stones placed around the foundation) is the most common, though the article says such countermeasures are temporary because the stones can move or be washed away in a flood, and that the US Federal Highway Administration recommends designing pier foundations that resist scour without relying on riprap at all. The practical lessons are to found piers deep, to inspect the river bed after floods, and to watch the depth under each pier.

Cables: Corrosion You Cannot See

The Forth Road Bridge's main cables (How Bridges Work 7) are made of 11,618 steel wires each, wrapped and hidden. The article on the bridge gives this sequence: an inspection programme began in 2003 (costing £1.2 million) after corrosion had been found in older US suspension bridges of similar design; in 2005, potentially serious structural issues were discovered; and when the study was completed two years later it reported an **8% to 10% loss of strength**. The worst-case projection was that traffic restrictions might be needed by 2014 and full closure by 2020. The response was to keep the air among the strands **below 40% humidity** with a dehumidification system: work by Faber Maunsell began in 2006, took two and a half years and cost £7.8 million, and some corroded strands were spliced. Acoustic monitoring for wire breaks was commissioned in August 2006.

How serious is a loss of 8% to 10%? That depends on the margin the cables started with. Using the 581 MPa wire stress of How Bridges Work 7 and an *assumed* breaking strength of 1,500 MPa:

```
WIRES = 11618
WORKING_MPA = 581                       # wire stress from How Bridges Work 7
BREAK_MPA = 1500                        # assumed breaking strength of the wire

def factor(loss):
    """Safety factor when a fraction `loss` of the strength is gone."""
    return (1 - loss) * BREAK_MPA / WORKING_MPA

for loss in (0, 0.05, 0.08, 0.10, 0.15, 0.20, 0.30):
    print(loss, round(factor(loss), 2))
# 0 2.58
# 0.05 2.45
# 0.08 2.38
# 0.1 2.32
# 0.15 2.19
# 0.2 2.07
# 0.3 1.81

for target in (2.0, 1.5):
    lost = 1 - target * WORKING_MPA / BREAK_MPA
    print(target, round(lost * 100, 1), round(lost * WIRES))
# 2.0 22.5 2618
# 1.5 41.9 4868
```

With these assumptions the cables begin with a factor of 2.58 and still have 2.32 after a 10% loss, so there was no immediate danger, but the loss would grow if nothing were done. The factor falls to 2.0 at a loss of 22.5% (about 2,618 wires' worth) and to 1.5 at 41.9%. The worst case did not arrive. Instead the problem was found by inspection, stopped by dehumidification and watched by acoustic monitoring. It is a good example of the kind of failure that *does not happen* because of regular, expensive, unglamorous work. (The relative loss figures are the article's. The 1,500 MPa strength and the resulting factors are my assumptions.)

What Each Failure Added to the Checklist

EVENT	WHAT WAS MISSED	WHAT THE CHECKLIST GAINED
Tay Bridge, 1879	wind pressure; weak cast-iron lugs; maintenance (the inquiry: "badly designed, badly constructed and badly maintained")	higher wind pressures (the Board of Trade later required 56 lb/sq ft); more care with brittle details
Tacoma Narrows, 1940	aerodynamic stability of a slender deck	wind-tunnel testing and aerodynamic shaping of decks (the article says the collapse boosted bridge aerodynamics research)
Scour failures	the river bed moving beneath the foundations	deep foundations, flood inspections, scour countermeasures
Forth Road Bridge cables, 2005	corrosion hidden inside the cable	inspection of hidden parts, dehumidification, acoustic monitoring

The pattern is that each disaster widened the list of things engineers must check: wind, then wind *dynamics*, then water, then time. Several of these cases (the Tay Bridge, Tacoma Narrows, the Forth Road Bridge cables) appeared earlier in this course in other roles, and they belong together here as one story: a bridge is safe only against the loads somebody thought of.

Common mistakes

Calling every wind failure "resonance" (flutter is a different mechanism); treating a safety factor as protection against effects that were never in the calculation (it covers uncertainty in the loads you know, not loads you omitted); inspecting only what can be seen (cables, foundations and bearings

hide their damage); and trusting a single dramatic number (the Tacoma film is shown too fast; the percentage loss in the cables needs its starting margin to mean anything).

Where This Leads

That is every tool of the course. The final chapter puts them to work: choose a bridge type for a site, estimate the loads with a small program, plan the inspection and maintenance that keep it safe, and compare the three Forth crossings side by side.

EXERCISE 1

A deck mode has a frequency of 0.2 Hz. (a) Under a steady periodic push of 0.1 m/s^2 at exactly that frequency, the peak swing is F / (2ζω^2). Find it for damping ratios of 0.005, 0.01 and 0.02. (b) Suppose the wind adds negative damping a = kU, with k chosen so that a equals the structure's own damping 2ζω at U = 15.6 m/s (35 mph) for ζ = 0.01. For wind speeds of 12, 15.6, 17.9 and 22 m/s, find the net growth rate a/2 − ζω and the doubling time when it is positive. Which of the two effects levels off, and which does not?

View solution

EXERCISE 2

A friction pile of diameter 1.0 m carries 1000 kN, with a soil skin friction of 60 kPa over its embedded length. For pile lengths of 10, 12, 15 and 18 m find the safety factor with no scour, the scour depth at which it falls to 1.5, and the scour depth at which it falls to 1.0. Then repeat the last two for 12 m piles carrying 1500 kN. What does the result say about the effect of load on the scour a foundation can stand?

View solution

EXERCISE 3

A cable of 11,618 wires has a breaking strength of 1,500 MPa (assumed). Its working stress is 581 MPa. Print the factor of safety for strength losses of 0, 5, 10, 20, 30 and 40%. Find the loss at which the factor falls to 2.0 and to 1.5, and the equivalent number of wires. Then repeat for working stresses of 450 and 700 MPa: which design has the most margin, and why does this matter when you are told "8% to 10% of the strength has gone"?

View solution

QUICK REFERENCE

- **Tacoma Narrows (1940):** opened 1 July, collapsed 7 November; a 39 ft wide deck on a 2,800 ft span; aeroelastic flutter, self-exciting and unbounded
- **Resonance:** an outside rhythm; limited by damping. **Flutter:** the wind's force depends on the motion; unbounded above a critical speed
- **Scour:** water removes the soil round foundations; estimated 60% of failures are scour and other hydraulic causes
- **Riprap:** the most common countermeasure, but temporary; better to found piers to resist scour
- **Forth Road Bridge cables:** 8% to 10% strength loss reported; dehumidification keeps air below 40% humidity
- **Pattern:** every failure adds a load or an effect to the engineer's checklist

COMING NEXT

How Bridges Work 10 is the capstone: choosing a bridge type for a site, estimating loads with a small program, planning inspection and maintenance, and comparing the three Forth crossings.

How Bridges Work

Chapter 10 · Capstone: Design and Test a Bridge

Nine chapters have given you the pieces: loads and forces, beams, arches, trusses, materials, cantilevers, suspension, cable stays, and what goes wrong. This chapter puts them together in the order a designer would meet them: a site, a type, the loads, the sizes, the checks against failure, the maintenance, and finally a look back at the three Forth crossings with the tools you now have. Everything here is built from functions you have already seen, and every number in a model is labelled as an assumption or as a figure from a named source.

> **WHAT WAS RUN, AND WHAT TO TRUST**
>
> Every Python snippet in this chapter and in the three exercise solutions was executed, and the output shown in the comments is what Python printed. These are **teaching models**, not design methods: they show how the pieces fit, and a real design needs a design code, an experienced engineer and checks well beyond these. Figures about the three Forth bridges come from the sources named in How Bridges Work 6 to 8.

Step 1: Choose a Type

The first question is the span: how wide is the gap? Each structure type in this course has a range where it makes sense. The evidence points are all in earlier chapters: a stone slab spans a few metres (How Bridges Work 3, about 5.6 m in the example); a plain steel beam can support only itself up to about 43 m (How Bridges Work 2, and far less with traffic); the Forth Bridge's cantilevers and girder span 521 m (How Bridges Work 6); the Queensferry Crossing's longest span is 650 m (How Bridges Work 8); and the Forth Road Bridge's suspension span is 1,006 m (How Bridges Work 7). A small function turns these into a rule of thumb:

```
def suggest(span_m):
    """A teaching rule of thumb from this course's examples, not a design code."""
    if span_m <= 10:
        return "slab or simple beam"
    if span_m <= 40:
        return "steel or concrete girder"
    if span_m <= 200:
        return "truss, arch or deep girder"
```

```
    if span_m <= 700:
        return "cable-stayed, cantilever or truss"
    return "suspension (or a very long cable-stayed span)"

for span in (6, 25, 120, 300, 520, 650, 1006):
    print(span, suggest(span))
# 6 slab or simple beam
# 25 steel or concrete girder
# 120 truss, arch or deep girder
# 300 cable-stayed, cantilever or truss
# 520 cable-stayed, cantilever or truss
# 650 cable-stayed, cantilever or truss
# 1006 suspension (or a very long cable-stayed span)
```

The thresholds are my own, drawn from the course's examples (the 700 m limit, for example, sits just above the 650 m of the Queensferry Crossing, and the real overlaps are wide). Span is only the first filter. The others come from the site:

SITE QUESTION	WHY IT MATTERS	WHERE THE COURSE COVERED IT
What is the ground like?	an arch or a suspension bridge pushes sideways and needs rock or a huge anchorage; a beam only pushes down	How Bridges Work 3 and 7
How much clearance is needed?	ships need height and an open channel; piers in the water cause scour	How Bridges Work 9
How exposed is it to wind?	a slender deck can flutter; a stiff truss or box resists	How Bridges Work 9
How will it be built?	balanced cantilever needs no falsework in the water; an arch needs centring	How Bridges Work 3, 6 and 8
How will it be maintained?	hidden cables and foundations need a plan for inspection	How Bridges Work 5 and 9

Step 2: Estimate the Loads

How Bridges Work 1's loads (dead, live and environmental) become numbers. Take a two-lane road deck 12 m wide (assumed) with a 0.25 m concrete slab (How Bridges Work 1's example),

surfacing and barriers of 12 kN per metre of length (assumed), and the round live load of 5 kN per square metre used in How Bridges Work 1:

```
width = 12                                  # m, a two-lane road with edge barriers (assum
slab = 0.25 * width * 24                    # kN/m: 0.25 m of concrete at 24 kN/m3
surfacing = 12                              # kN/m: surfacing and barriers (assumed)
dead = slab + surfacing
live = 5 * width                            # kN/m: 5 kN on every m2, as in How Bridges Wo
print(dead, live, dead + live)
# 84.0 60 144.0
```

The deck load is 144 kN per metre of bridge: 84 kN/m dead and 60 kN/m live. Wind, temperature and water (the environmental loads) are not in this figure, and for a long, slender bridge they can control the design (How Bridges Work 8 and 9). The 144 kN/m is the number the rest of this chapter's models use.

Step 3: Size the Main Elements

For a short span, the main elements are girders. Here is the beam calculation from How Bridges Work 2 applied to a 25 m road span, carried by six equal steel I-girders, trying deeper sections one at a time until one passes both the stress limit (165 MPa) and a sag limit of span/360, with each girder's own weight added:

```
import math

E, ALLOW = 200e3, 165                       # MPa
RHO, G = 7850, 9.81

def i_section(bf, tf, tw, depth):
    inner = depth - 2 * tf
    return (bf * depth**3 - (bf - tw) * inner**3) / 12

def area_i(bf, tf, tw, depth):
    return 2 * bf * tf + tw * (depth - 2 * tf)

SECTIONS = [(250, 16, 10, 600), (300, 20, 12, 800), (350, 22, 14, 1000), (400, 25, 16, 1
```

```
def check(span_m, w_load, girders):
    """Try each section in turn on one of `girders` equal girders; return the first that
    for bf, tf, tw, d in SECTIONS:
        w = w_load / girders + RHO * G * area_i(bf, tf, tw, d) * 1e-6 / 1000   # one gir
        i = i_section(bf, tf, tw, d)
        m = w * span_m**2 / 8
        stress = m * 1e6 * (d / 2) / i
        sag = 5 * w * (span_m * 1000) ** 4 / (384 * E * i)
        limit = span_m * 1000 / 360
        ok = stress <= ALLOW and sag <= limit
        print(d, round(w, 1), round(stress, 1), round(sag, 1), round(limit, 1), ok)
        if ok:
            return d
    return None

print(check(25, 144, 6))
# 600 25.1 703.2 763.1 69.4 False
# 800 25.6 353.6 287.8 69.4 False
# 1000 26.2 217.8 141.8 69.4 False
# 1200 27.0 141.5 76.8 69.4 False
# 1400 27.8 98.1 45.6 69.4 True
# 1400
```

Each line is the depth (mm), the load on one girder including its own weight (kN/m), the stress (MPa), the sag (mm), the sag limit (mm) and the verdict. Look at the 1,200 mm girder: its stress (141.5 MPa) is within the limit, but it sags 76.8 mm against 69.4 mm allowed, so it fails. **Stiffness decided**; the 1,400 mm girder passes with a stress of only 98.1 MPa. This is the lesson of How Bridges Work 2 again: a beam is often governed by how much it sags, not by whether it breaks.

For long spans the main element changes. The code below compares the steel in the *main load-carrying system* of a truss and of a suspension cable, per metre of bridge, with both carrying the 144 kN/m deck load plus their own weight. Each repeats its calculation until its own weight settles, or gives up if the weight keeps spiralling. The truss has a constant depth of span/10 (assumed) and the stress limit used in How Bridges Work 2 (165 MPa); the cable has a sag of span/10 (assumed) and the 600 MPa used in How Bridges Work 7. A factor of 1.5 on the truss chords stands for bracing and joints:

```
W_DECK = 144                              # kN/m: deck, surfacing and traffic

def truss_steel(span, depth_ratio=10, sigma_mpa=165, extra=1.5):
    """Steel (t per metre) for a truss of depth span/depth_ratio, found by repeating
    the calculation until its own weight stops changing. None if it never settles."""
    w = W_DECK
    for _ in range(500):
        moment = w * span**2 / 8                      # kN m
        force = moment / (span / depth_ratio)         # chord force, kN
        area = extra * 2 * force / (sigma_mpa * 1e3)  # m2 (kN / kPa), both chords plus l
        w_new = W_DECK + area * RHO * G / 1000
        if abs(w_new - w) < 1e-6:
            return area * RHO / 1000
        w = w_new
        if w > 1e6:
            return None
    return None

def cable_steel(span, sag_ratio=10, sigma_mpa=600):
    """Main-cable steel (t per metre), same method. Towers and anchorages are not counte
    w = W_DECK
    for _ in range(500):
        h = w * span**2 / (8 * span / sag_ratio)
        v = w * span / 2
        area = math.hypot(h, v) / (sigma_mpa * 1e3)
        w_new = W_DECK + area * RHO * G / 1000
        if abs(w_new - w) < 1e-6:
            return area * RHO / 1000
        w = w_new
        if w > 1e6:
            return None
    return None

for span in (50, 100, 200, 300, 500, 800, 1200):
    t = truss_steel(span)
    c = cable_steel(span)
    print(span, None if t is None else round(t, 2), None if c is None else round(c, 2))
# 50 1.41 0.13
# 100 3.11 0.26
# 200 7.91 0.53
# 300 16.23 0.8
# 500 102.84 1.39
# 800 None 2.35
```

```
  # 1200 None 3.84
```

The columns are the span (m), the truss steel (tonnes per metre) and the cable steel (tonnes per metre). Read the *trend*. Going from 50 m to 300 m (six times the span), the cable's steel rises about six times (0.13 to 0.8), roughly in step with the span, but the truss's rises more than eleven times (1.41 to 16.23), and at 500 m it has ballooned to 102.84 and by 800 m there is no solution at all: the truss's own weight needs more steel than it can carry. This is the plain beam's span limit of How Bridges Work 2, pushed out by depth but not removed. **Be careful with the comparison.** The cable figure counts only the main cables. The towers, anchorages, hangers and the deck they carry are all missing, and the truss figure includes the whole load-carrying system. A constant-depth truss is also pessimistic: the Forth Bridge reaches 521 m by tapering its depth to follow the moment (How Bridges Work 6). What the numbers do show is how the two systems *scale*: a long span needs a system where load travels in tension.

Step 4: Test Against What Can Go Wrong

CHECK	QUESTION	CHAPTER
Bending and shear	does every member carry its moment and shear within the stress limit?	2
Sag	is the deck stiff enough (span/360 or whatever the code sets)?	2, 6
Buckling	can any compression member bow out sideways?	4
Abutments and anchorages	can the ground take the thrust or the cable pull without sliding or tipping?	3, 7
Fatigue	will repeated traffic cycles crack a detail within the design life?	5, 9
Wind and flutter	is there a wind speed at which the deck feeds itself energy?	8, 9
Scour	what is left of the foundation after a flood lowers the river bed?	9
Corrosion	what is the margin if some strength is lost, and can you inspect it?	5, 9

Step 5: Plan Inspection and Maintenance

A bridge that is never inspected is a bridge waiting for a surprise. A maintenance plan lists each element, how often it is inspected or renewed, and what each event costs. The model plan below uses illustrative intervals and costs (the 1,200 thousand pound figure for a cable interior inspection echoes the £1.2 million quoted for the Forth Road Bridge's 2003 inspection programme in How Bridges Work 9, but the other figures are invented for the example):

```
# element: (inspection interval in years, cost of each inspection in £ thousand), illust
PLAN = {
    "bearings and expansion joints": (2, 15),
    "coating condition survey": (5, 40),
    "coating renewal": (25, 3000),
    "cable interior inspection": (10, 1200),
    "scour survey of piers": (1, 20),
    "fatigue-prone details": (6, 100),
}

def schedule(years, plan):
    totals = {}
    for name, (every, cost) in plan.items():
        count = years // every
        totals[name] = (count, count * cost)
    return totals

result = schedule(50, PLAN)
for name, (count, cost) in result.items():
    print(f"{name:32} {count:3} {cost:6}")
print(sum(c for _, c in result.values()))
# bearings and expansion joints     25    375
# coating condition survey          10    400
# coating renewal                    2   6000
# cable interior inspection          5   6000
# scour survey of piers             50   1000
# fatigue-prone details              8    800
# 14575
```

Over 50 years the plan costs 14,575 thousand pounds in this model, and **two items make up 82%**: renewing the coating twice and inspecting the cable interior five times. Cheap, frequent checks (the yearly scour survey at 20 thousand) add up to far less than a single big job, but they are the ones that find a problem early. Real bridge owners balance both. How Bridges Work 5

gave the Forth Bridge as the example: a coating designed to last about 25 years, a ten-year refurbishment that cost £130 million and finished in December 2011.

Step 6: The Three Forth Crossings, Side by Side

With the tools you now have, the three bridges that run through this course make a complete case study. The figures below are those from How Bridges Work 6 to 8, from the sources named there. The last column of the code gives the years since the 1890 Forth Bridge opened:

```
# (name, opened, type, longest span m, total length m, tower height m) from How Bridges
BRIDGES = [
    ("Forth Bridge", 1890, "cantilever", 521, 2467, 110),
    ("Forth Road Bridge", 1964, "suspension", 1006, 2512, 156),
    ("Queensferry Crossing", 2017, "cable-stayed", 650, 2700, 207),
]
base = BRIDGES[0]
for name, year, kind, span, length, tower in BRIDGES:
    print(f"{name:21} {kind:13} {span:5} {span / base[3]:.2f} {length:5} {tower:4} {year
# Forth Bridge          cantilever      521 1.00  2467  110    0
# Forth Road Bridge     suspension     1006 1.93  2512  156   74
# Queensferry Crossing  cable-stayed    650 1.25  2700  207  127
```

The columns are the name, the type, the longest span (m), the span relative to the Forth Bridge, the total length (m), the tower height (m) and the years after 1890. The suspension bridge's main span is 1.93 times the cantilever's; the cable-stayed bridge's 650 m is 1.25 times. (The tower heights are measured to different references in the sources: above high water for the Forth Bridge, above mean water level for the Forth Road Bridge, and a plain height for the Queensferry Crossing, so compare them only loosely.)

	FORTH BRIDGE (1890)	FORTH ROAD BRIDGE (1964)	QUEENSFERRY CROSSING (2017)
Load path	deep steel arms balanced over three towers; tension on top, compression below	deck hangs from a draped main cable, which runs to towers and anchorages	straight stays from three towers to the deck, which is in compression

	FORTH BRIDGE (1890)	FORTH ROAD BRIDGE (1964)	QUEENSFERRY CROSSING (2017)
Material story	the first major steel structure in Britain; painted against corrosion; coating renewed to a 25-year design in 2011	high-tensile steel wire cables of 11,618 wires each	deck segments built in China and Spain; stays crossing at midspan
What went wrong or nearly did	the coating's never-ending job; Bouch's earlier suspension design was abandoned after the Tay Bridge collapse	cable corrosion found in 2005; 8% to 10% strength loss reported	25 days lost to wind in 2016; ice falling from towers closed it in 2020
Lesson	depth where the moment is largest	tension is the efficient way to cross a long gap, but hidden steel must be inspected	the wind and the weather shape the design, in the building and in use

Common mistakes

Choosing a type from the span alone (ground, wind, clearance and maintenance matter just as much); trusting a model's trend as if it were a design (the truss-against-cable table leaves out towers, anchorages and the deck); checking strength but not stiffness (the 1,200 mm girder failed on sag); and designing without a maintenance plan (the cheapest bridge to build is not always the cheapest to own).

EXERCISE 1

Using `truss_steel` and `cable_steel` from this chapter, find (a) the longest span, in steps of 10 m, for which the truss calculation still settles on an answer, (b) the shortest span, in steps of 10 m, at which the truss needs more than 20 times the steel of the cable, and (c) the truss-to-cable ratio at 50, 100, 200 and 300 m. What does the trend say about how steel use grows with span in the two systems, and what does the model leave out?

View solution

EXERCISE 2

A 35 m road span carries a deck load of 144 kN/m shared by six identical steel I-girders. Try the sections (350, 22, 14, 1000), (400, 25, 16, 1200), (450, 28, 18, 1400), (500, 32, 20, 1600) and (550, 36, 22, 1800) in turn (flange width, flange thickness, web thickness, depth, in mm), adding each girder's own weight.

Use a stress limit of 165 MPa and a sag limit of span/360. Print the result for each, report the first that passes, and say which limit decided the failures.

View solution

EXERCISE 3

Using the maintenance plan of this chapter, print for a 50-year life the number of events, the cost and the percentage share of each element. Then change the coating renewal interval from 25 to 40 years and the cable interior inspection from every 10 years to every 5, and print the new total and the effect of each change alone. Which element dominates the budget, and which change matters most?

View solution

QUICK REFERENCE: THE WHOLE COURSE

- **1 Forces and Loads:** five forces; dead, live and environmental loads; the load path; reactions; stress
- **2 Beams and Simple Spans:** shear and moment; stress = My/I; depth counts cubed; sag grows with the fourth power of the span
- **3 Arches and Early Bridges:** stone in compression; thrust $H = wL^2/(8f)$; abutments resist sliding and tipping
- **4 Trusses:** triangles; tension and compression bars; method of joints; buckling
- **5 Materials:** compression and tension strengths; fatigue; corrosion; paint as maintenance
- **6 Cantilevers and the Forth Bridge:** balanced arms over towers; the moment peaks at the tower
- **7 Suspension Bridges:** the cable as a parabola; the sag controls the pull; anchorages
- **8 Cable-Stayed Bridges:** stays in tension push the deck into compression; fan and harp; wind pressure rises with speed squared
- **9 Wind, Water and Failure:** flutter is not resonance; scour removes the foundation's ground; hidden corrosion needs inspection
- **10 Capstone:** choose, load, size, test, maintain

COURSE COMPLETE

That is the end of *How Bridges Work*. If you walk or drive across any of the three Forth bridges, you can now say which way the load goes from the road to the ground, which members are pulling and which pushing, and what the inspectors are looking for.